Economics of incomplete information

Traditional microeconomic analysis deals with economic agents making decisions under complete information.

Examples of such assumptions:
• Consumers know the utility they get from a good;
• Firms know demand schedules;
• Firms know each others’ prices; and so on.

Real life is more complicated and less certain.
Comparing projects with uncertain outcomes

Every project has two characteristics, the expected value and the degree of risk.

A certain outcome = no risk.

The expected value, or the mean:
Computation as the weighted sum of all possible payoffs (“weighted” = multiplied by the probabilities of each respective outcome):

\[ E[x] = q_1x_1 + q_2x_2 + \ldots + q_nx_n, \]

where \( x_i \) is payoff \( i \), \( q_i \) is the probability that payoff \( i \) occurs, and \( q_1 + q_2 + \ldots + q_n = 1 \).
**Variance** (a measure of risk):

The sum of (the probabilities of each outcome multiplied by the squared differences between the value of the random variable and its mean:

$$\text{Var} = q_1(x_1 - E[x])^2 + q_2(x_2 - E[x])^2 + \ldots + q_n(x_n - E[x])^2$$

The standard deviation, $\sigma$, is the square root of the variance.

The larger the variance (or the standard deviation), the riskier the project.
(For events occurring with certainty, $\text{Var} = 0$)
How much would you be willing to pay for a lottery ticket that pays $100 with a 50% probability and nothing with a 50% probability?

I personally would pay… $30

What is the expected value of this lottery?

\[ E[x] = q_1x_1 + q_2x_2 = 0.5 \cdot 100 + 0.5 \cdot 0 = $50 \]

Why the difference?
The attitude to risk may vary.

• Individuals who prefer less risk to more risk, all other things being equal, are called “risk averse”. It is believed that most people belong to this group.

• Those who don’t care about the degree of risk and care only about the expected value are called “risk neutral”.

• Individuals who prefer more risk to less risk, all other things being equal, are called “risk preferring”, or “risk loving”.
  (Sort of an anomaly.)

In the example above, the person in question is . . . risk averse.
“Risk premium” – the minimum reward that would induce a risk averse person to accept risk while preserving the same expected value.

Alternatively, risk premium is the maximum amount of money an individual will be willing to pay to replace an uncertain situation with a certainty situation that has the same expected value.

Risk premium depends on the characteristics of the lottery as well as on individual preferences.

In the example above, my risk premium is $20.

I will pay only $30 for a lottery; in other words, will trade certain $50 for this lottery only for a premium of 50 – 30 = $20
Project selection

Suppose we have two projects:

A: expected value = $1000, Var = 5000
B: expected value = $1000, Var = 1500

Which one will each type choose?

Risk averse – will choose B
Risk neutral – indifferent; either A or B
Risk loving – will choose A
What if the expected values differ as well?

A: expected value = $1200, Var = 5000

B: expected value = $1000, Var = 1500

Everyone prefers higher expected value to lower expected value, all other things being equal. Risk preferences of each type are the same as on the previous slide.

<table>
<thead>
<tr>
<th>Type of individual</th>
<th>Preferences</th>
<th>Exp. value</th>
<th>Risk</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk averse</td>
<td>A</td>
<td>B</td>
<td>It depends</td>
<td></td>
</tr>
<tr>
<td>Risk neutral</td>
<td>A</td>
<td>either</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Risk loving</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>
What can a risk averse individual do to reduce risk?

1. Be informed.

   Useful information has value!!!

2. Diversify.
Diversification, or “spreading the risk”.

You are considering investing $100 into one or two assets (stocks, for concreteness).

Each stock is worth $50 now, and you believe that within the next three months its value can with equal probability either increase to $80 or drop to $40.

For now, let us assume that what happens to one stock is not correlated to what happens to the other one.
Expected value of the investment:

If you invest in the shares of only one of the companies:

\[ E[x] = 2 \times (0.5 \times 80 + 0.5 \times 40) = $120 \]

If you buy one share of each of the two companies:

\[ E[x] = (0.5 \times 80 + 0.5 \times 40) + (0.5 \times 80 + 0.5 \times 40) = $120 \]
The variance:

If you invest in one company:

$$\text{Var} = 0.5 \times (160 - 120)^2 + 0.5 \times (80 - 120)^2 =$$

$$= 0.5 \times 1600 + 0.5 \times 1600 = 1600$$

If you invest in both…

Possible outcomes:

W/prob $\frac{1}{4}$ both stocks go up, $x = $160
W/prob $\frac{1}{4}$ stock A goes up, stock B falls, $x = $120
W/prob $\frac{1}{4}$ stock B goes up, stock A falls, $x = $120
W/prob $\frac{1}{4}$ both stocks fall, $x = $80

$$\text{Var} = 0.25 \times (160 - 120)^2 + 0.5 \times (120 - 120)^2 + 0.25 \times (80 - 120)^2 =$$

$$= 0.25 \times 1600 + 0.25 \times 1600 = 800$$
If the payoffs from two assets are negatively correlated, then diversification becomes even more attractive.

Example: Two companies compete for a large government contract. After the winner is announced, the winner’s stock goes up and the loser’s stock falls.

When firms undertake many projects at the same time, it is best for them to be risk neutral.

Moreover, shareholders WANT managers to act in a risk-neutral manner (to care only about expected values).
Summary:

• Everyone prefers a higher expected value to a lower expected value.

• Most individuals are risk averse. This means they prefer less risk to more risk, provided the expected value stays the same. If one project offers a higher expected value and a higher risk at the same time, then we need more information to tell which of the two will a risk averse person choose.

• Firms can be assumed to be risk neutral. They evaluate projects based solely on their expected values.
Consider a modification of problem 4 on p.469.

You are the manager of a firm that sells soybeans in a perfectly competitive market. Your cost function is $C(Q) = 2Q + 2Q^2$. Due to production lags, you must make your output decision prior to knowing what the market price is going to be. You believe that there is a 25% chance the market price will be $120 and a 75% chance it will be $160. What is the optimal quantity of output?

The good news: All the rules we have learned before (MR=MC, etc.) still apply but “expected” appear in them as needed.

Let’s do it step by step.
Normally, the rule we’d apply would be P = MC. Here, we replace P with its expected value, E(P).

a. Calculate the expected market price.

\[ E(P) = 0.25 \cdot 120 + 0.75 \cdot 160 = 30 + 120 = 150 \]

b. What output should you produce to maximize expected profits?

\[ E(P) = MC \quad \text{TC} = 2Q + 2Q^2 \quad \text{therefore} \quad MC = 2 + 4Q \]

\[ 150 = 2 + 4Q \]
\[ 148 = 4Q \]
\[ Q = 37 \]
c. What are your profits under each outcome and the expected profits?

You produce $Q=37$ which determines your cost,
\[ TC = 2 \cdot 37 + 2 \cdot 37^2 = 74 + 2738 = \$ 2,812 \]

If $P = 120$, your profit is \[ 120 \cdot 37 - 2812 = \$ 1,628 \] (happens w/prob $\frac{1}{4}$)

If $P = 160$, your profit is \[ 160 \cdot 37 - 2812 = \$ 3,108 \] (happens w/prob $\frac{3}{4}$)

Expected profit = \[ \frac{1}{4} \cdot 1628 + \frac{3}{4} \cdot 3108 = \$ 2,738 \]
Looks like in one case we are underproducing and in the other case – overproducing. Wouldn’t it be better to bet on the most likely outcome?

\[ P = 160 \quad \text{MC} = 2 + 4 \, Q \]

\[ 4 \, Q = 158 \]

\[ Q = 39.5 \quad \text{and} \quad TC = 2 \cdot 39.5 + 2 \cdot 39.5^2 = \$ 3,199.50 \]

If \( P = 160 \), our profit = \( 160 \cdot 39.5 - 3199.50 = \$ 3,120.50 \)

If \( P = 120 \), our profit = \( 120 \cdot 39.5 - 3199.50 = \$ 1,540.50 \)

Expected profit = \( \frac{1}{4} \cdot 1540.5 + \frac{3}{4} \cdot 3120.5 = \$ 2,725.50 \)
The same approach can be extended to the imperfectly competitive market case. Consider the following problem:

A firm with market power produces at constant marginal (and average) cost of $1. There is a 50% chance of a recession and a 50% chance of an economic boom.

During a boom, the inverse demand for firm’s product will be

\[ P = 10 - 0.5 Q \]

If there is a recession, the inverse demand will be

\[ P = 6 - 0.5 Q \]

The firm is risk neutral and must set output before demand is known. How much output should it produce to maximize expected profit?
Normally, we would look for the point where $\text{MR} = \text{MC}$. This time, we will do $\text{E(MR)} = \text{MC}$.

There are two equally good ways to find expected marginal revenue:

1. Find expected demand, then expected marginal revenue:

   Expected inverse demand:
   \[
   \text{E(P)} = 0.5 (10 \,- \, 0.5 \, Q) \, + \, 0.5 \, (6 \, - \, 0.5 \, Q) \, = \, \ldots \, = \, 8 \, - \, 0.5 \, Q
   \]
   \[
   \text{E(MR)} \, = \, 8 \, - \, Q
   \]

2. Find the marginal revenue under each scenario, then find the expected MR:

   Boom: \quad P = 10 \,- \, 0.5 \, Q \quad \text{MR} = 10 \, - \, Q
   
   Recession: \quad P = 6 \, - \, 0.5 \, Q \quad \text{MR} = 6 \, - \, Q

   \[
   \text{E(MR)} \, = \, 0.5 \, (10 \, - \, Q) \, + \, 0.5 \, (6 \, - \, Q) \, = \, 8 \, - \, Q
   \]
The rest is trivial:

\[ E(MR) = 8 - Q \quad \text{MC} = 1 \]

\[ E(MR) = MC \]

\[ 8 - Q = 1 \quad Q = 7 \]

If boom, then

\[ P = 10 - 0.5 \cdot Q = 6.50 \]

Profit = \( (P - AC) \cdot Q = (6.50 - 1) \cdot 7 = 38.50 \)

If recession, then

\[ P = 6 - 0.5 \cdot Q = 2.50 \]

Profit = \( (P - AC) \cdot Q = (2.50 - 1) \cdot 7 = 10.50 \)

Expected profit = \( 0.5 \cdot 38.50 + 0.5 \cdot 10.50 = 24.50 \)

Or directly:

“Expected price” given \( Q = 7 \) is \( E(P) = 8 - 0.5 \cdot Q = 4.50 \)

Exp.profit = \( (E(P) - AC) \cdot Q = (4.50 - 1) \cdot 7 = 24.50 \)
Consumer search for the best price and implications for the firm’s behavior

General idea:
A consumer samples several stores and obtains a price quote from each. The cost of obtaining each quote is the same. The total number of stores is large, so “drawing” one of them doesn’t affect the odds.

After several quotes, you can always return to the store with the best price.

It makes sense to continue searching as long as the (expected) benefit exceeds the cost of search.
Expected benefit:

Joe wants to buy a DVD player. He thinks one-third of the stores charge $130 for a DVD player, one-third charge $100, and one-third charge $85. He sampled one store and the price was $100. What is the expected benefit from sampling another store?

w/ prob 1/3 next P = $85, a $15 benefit

w/ prob 1/3 next P = $100, no benefit

w/ prob 1/3 next P = $130, since he can return to the first store, no benefit

Exp. benefit = (1/3)·15 = $5
How will the answer change if the best price found so far is $130?

w/prob 1/3 next P = $85, a $45 benefit

w/prob 1/3 next P = $100, a $30 benefit

w/prob 1/3 next P = $130, no benefit

Exp.benefit = (1/3)·45 + (1/3)·30 = $25
In general, if you sample a store and the price is high, the expected benefit from further search is greater (it makes more sense to keep searching).

The lower the observed price, the more sense it makes to stop searching and buy.

This principle holds even if the distribution of prices is not known.
Cost/benefit of continuing to search

Exp. benefit of another search

Cost of another search

Price observed

If observed $P$ is at or below this level, we stop searching and buy.
What happens if the cost of search increases?

Consumers are more likely to settle for higher prices.
What has happened with the advent of the Internet?

Search cost decreased; As a result, consumers buy same goods at lower prices

Are all industries affected equally?

(Durable) consumer goods – very much so,
Groceries and expendable household items – less;
Travel fares – affected a lot;
Industrial shipping rates – less. Why?
Insurance rates, phone rates – also affected a lot
Socially optimal risk sharing

If individuals are risk averse while firms are risk neutral, what is the optimal risk sharing between consumers and firms?

Getting out of risk has more value for consumers than for firms.

Therefore there is room for mutually beneficial exchange, where consumers reward the firm for accepting some of their risk for them.

Example: insurance industry.
Insurance companies are able to make money selling insurance because they may
- have better information about the odds than their clients;
- differ from clients in their attitude to risk;
- diversify.

The larger the group of the insured, the **smaller**
the variance, hence the **lower** the risk.