

# **Firm Value And The Debt-Equity Choice**

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## ***Abstract***

*Building on the no growth perpetuity framework first developed by Modigliani and Miller (1963), this paper attempts to offer gain to leverage ( $G_L$ ) formulations useable by managers in making debt-equity choices. These formulations focus on how changes in equity and debt discount rates influence firm value. A real world application (using data suggested by independent analysts) seeks to determine the gain to leverage for different debt-equity choices. Using our formulation with constant growth, we offer results that can support the suggested target debt-equity choice as the choice that maximizes firm value.*

## **I. Introduction and Background**

According to Compustat, since the beginning of the century there have been about 1,650 firms per year that on average have reported no long-term debt (which includes capitalized lease obligations). Gopalakrishnan (1994) indicates about 30 percent of such unlevered firms will issue debt within a year and maintain it for a prolonged (if not permanent) period of time. However, larger firms without long-term debt are a rarity as shown by Agrawal and Nagarajan (1990) who find only 104 such firms listed on major U.S. stock exchanges. This suggests that most managers, at least for larger firms, behave as if value can be added by choosing some positive debt level when financing their operating assets.

Theoreticians offer various formulas to support the managerial decision to issue debt. The forerunner of this line of research is Modigliani and Miller (1963), referred to as MM. They derive a gain to leverage ( $G_L$ ) formulation in the context of an unlevered firm issuing risk-free debt to replace risky equity. For MM,  $G_L$  is the corporate tax rate multiplied by debt value. The applicability of MM's  $G_L$  formulation is limited as it implies that financial executives issue unrestricted amounts of debt. Extensions of MM consider a variety of leverage-related wealth effects (most noteworthy, the

effects stemming from personal tax, flotation costs, bankruptcy, agency, and asymmetric information considerations).

Empirical researchers offer differing opinions concerning the strength of leverage-related effects. While early researchers (Warner, 1977; Miller, 1977) suggest such effects may be unimportant (at least for larger firms), later investigators (Altman, 1984; Cutler and Summers 1988) contend otherwise indicating such effects would be significant if quantified. Graham and Harvey (2001) offer support for leverage-related effects but restrict this support by noting there is little evidence that executives are concerned about some effects (namely, personal taxes, transactions costs, asset substitution, free cash flows, and asymmetric information). Regardless of the significance of leverage-related effects, some researchers (Graham and Harvey, 2001; Pinegar and Wilbrecht, 1989) indicate that firms may be more concerned with an amount of debt that gives flexibility for future opportunities. Other researchers (Fischer, Heinkel, and Zechner, 1989; Kayhan and Titman, 2004) downplay the need for debt flexibility by offering evidence for the role performed by tax and bankruptcy cost effects. Hull (1999) presents event study evidence consistent with leverage-related effects determining an optimal debt level.

Given the presence of debt in the capital structure of most firms as well as the empirical evidence concerning leverage-related effects, there is a need to offer usable equations that can quantify these effects. This paper aims to fill this void by offering  $G_L$  formulations that quantify leverage-related wealth effects. This is done through perpetuity  $G_L$  formulations that make explicit how changes in equity and debt discount rates impact firm value. To the extent changes in such discount rates can be accurately estimated along with values for other relevant variables (such as growth and tax rates), the  $G_L$  formulations given in this paper can be used to measure the dollar impact of a proposed capital structure change. Consequently, it is possible for financial executives to make a debt-equity choice that maximizes firm value.

The remainder of the paper is organized as follows. Section II reviews the traditional  $G_L$  perpetuity formulations. Section III derives  $G_L$  formulations for an unlevered firm situation (although not shown in this paper, similar but lengthier formulations could be offered for firms that are already levered). Section IV gives computations for an application using real data. Section V reports the application's results for ten key variables for nine debt-equity choices. Section VI presents limitations of the application and Section VII gives summary statements.

## II. Traditional Perpetuity Gain to Leverage Formulations

This paper's  $G_L$  formulations are rooted in and developed within the no growth perpetuity framework of MM (1963) and Miller (1977). This section reviews these  $G_L$  formulations and their extensions. It ends by indicating the need to incorporate discount rates in  $G_L$  formulations.

### A. The MM Gain to Leverage Formulation

MM analyze the valuation impact of a debt-for-equity transaction. The simplifying conditions explicitly or implicitly used in their analysis include:

- (i) two security types (an unlevered firm with risky equity that issues risk-free debt);
- (ii) only corporate taxes (no personal taxes on income from either equity or debt);
- (iii) level perpetuities (which can approximate any series of unequal cash flows);
- (iv) no growth (depreciation each year equals investment to keep the same amount of capital);
- (v) no imperfections (i.e., no leveraged-related effects such as flotation costs, bankruptcy costs, agency effects, or asymmetric information effects); and,
- (vi) equivalent return classes (the CAPM had not yet been developed).

Given these conditions, MM argue that  $G_L$  is the exogenous corporate tax rate ( $T_C$ ) times the value of perpetual risk-free debt ( $D$ ) such that

$$G_L = T_C D. \quad (1)$$

$D$  is the chosen perpetual interest payment ( $I$ ) divided by the exogenous cost of capital on risk-free debt ( $R_F$ ). As  $D$  increases, MM posit that there is an increase in the rate at which risky equity is discounted. However, no quantitative application is made of any net negative impact on firm value of the increase in equity's discount rate. Similarly, no detailed valuation analysis is made of the  $G_L$  ramifications if debt is risky. However, if debt is risky, then we have

$$D = \frac{I}{R_D} \quad (2)$$

where  $R_D > R_F$  with  $R_D$  now an increasing function of debt.

While there are other forms of financing that might affect the debt-equity choice, little attention is given to these forms. For example, one form that might affect the choice is long-term lease financing. However, because any such lease payment acts like debt by lowering the firm's taxable income and increasing its financial risk, it resembles debt and can be treated as part of  $D$ . This is true for any off-balance-sheet items that behave like debt.

## *B. Extensions of the MM Formulation*

Those extending the  $G_L$  equation of MM (Baxter, 1967; Kraus and Litzenberger, 1973; Kim, 1978) assume risky debt. They argue that increasing debt levels are associated with increasing bankruptcy costs such that an optimal debt level exists where the negative bankruptcy costs effect offsets the positive tax shield effect. Increasing levels of debt can cause firm value to fall for reasons other than bankruptcy costs. For example, Jensen and Meckling (1976) examine a wider range of leverage-related costs that they call agency costs. Regardless of corporate tax shield and bankruptcy considerations, net agency effects can impact  $G_L$ . For example, increasing debt can initially cause net gains owing to the reduction in owner-manager monitoring costs, but can eventually lead to net losses due to the escalation in costs caused by restrictive debt covenants.

Drawing from the work of Farrar and Selwyn (1967), Miller (1977) assumes personal taxes and extends (1) such that

$$G_L = [1 - \alpha]D \quad (3)$$

where  $\alpha = \frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}$  with  $T_{PE}$  and  $T_{PD}$  the personal tax rates applicable to income from equity

and debt, and  $D$  now equals  $\frac{(1 - T_{PD})I}{R_D}$ . For Miller, costs related to the increase in debt (in particular,

bankruptcy costs) are inconsequential so that the effect of personal taxes alone offset the effect of corporate taxes. For example, Miller argues that  $\alpha \approx 1$ , and thus  $G_L \approx$  zero (e.g.,  $G_L = [1 - \alpha]D \approx [1 - 1]D \approx 0$ ). Regardless, as  $[1 - \alpha]$  in (3) takes on values smaller than  $T_C$ , then  $G_L$  in (3) becomes less than  $G_L$  in (1). Even if  $[1 - \alpha] = T_C$ ,  $G_L$  in (3) is less than  $G_L$  in (1) if  $T_{PD} > 0$  since  $D$  in (3) is

adjusted for personal taxes and now equals  $\frac{(1 - T_{PD})I}{R_D}$  instead of just  $\frac{I}{R_D}$ .

Even if Miller is correct, signaling theory (Leland and Pyle, 1977; Ross, 1977; Myers and Majluf, 1984) suggest that an increase in a firm's debt-to-equity ratio can lead to an increase in firm value. For example, Myers and Majluf (1984) argue that if managers are better informed than outside investors, firms are more likely to retire equity when it is undervalued. Thus, a debt-for-equity transaction would signal positive news in the sense underpriced securities are being retired (in addition to any other positive signal the firm conveys about its future cash flows covering larger

interest payments). Empirical research (Copeland and Lee, 1991; Hull and Michelson, 1999) offers evidence consistent with debt-for-equity transactions signaling positive news (including the conveyance of reduced risk as seen in lower betas and thus reduced required rates of return). In conclusion, signaling theory suggests exchanging debt for equity can cause  $G_L > 0$  to hold even if there are no other leverage-related effects.

Ensuing  $G_L$  extensions of MM (DeAngelo and Masulis, 1980; Kim, 1982; Modigliani, 1982; Ross, 1985) consider a variety of leverage-related costs and show that an optimal debt level exists even when personal taxes are recognized. Leland and Toft (1996) extend the closed-form results of Leland (1994) to a much richer class of possible debt structures permitting the study of the optimal amount of maturity of debt. Leland (1998) attempts to provide quantitative guidance on the amount and maturity of debt, the financial restructuring, and the optimal risk strategy. For the most part, the  $G_L$  extensions are characterized by the inability to make explicit how changes in equity and debt discount rates impact firm value within a model that financial managers might find useable.

### III. Formulations That Incorporate Discount Rates

In this section, practical  $G_L$  formulations are derived for managers making their debt-equity choices. These equations consider the impact of equity and debt discount rates for an unlevered firm for three situations: (i) no personal taxes and no growth, (ii) personal taxes and no growth, and, (iii) personal taxes and constant growth.

#### A. Gain to Leverage Formulation without Personal Taxes

A  $G_L$  formulation that includes discount rates can be derived from the definition that  $G_L$  is levered firm value ( $V_L$ ) minus unlevered firm value ( $V_U$ ). We have

$$G_L = V_L - V_U \quad (4)$$

where  $V_U$  and  $V_L$  are defined below and the general MM conditions described earlier hold.

$V_U$  is the same as unlevered equity value ( $E_U$ ).  $E_U$  is the uncertain perpetual after-corporate tax cash flow available to unlevered equity of  $(1-T_C)C$  divided by the exogenous unlevered equity discount rate ( $R_U$ ). We have

$$V_U = E_U = \frac{(1-T_C)C}{R_U} \quad (5)$$

where  $C$  is the perpetual before-tax cash flow available to unlevered equity owners with  $R_U > R_D$  if

the firm should choose to issue debt. Note that  $C$  assumes all expenses are cash expenses so that before-tax cash flow equals taxable income.

$V_L$  is levered equity value ( $E_L$ ) plus debt value ( $D$ ).  $E_L$  is the uncertain perpetual after-corporate tax cash flow available to levered equity of  $(1-T_C)(C-I)$  divided by the endogenous levered equity discount rate ( $R_L$ ). We have

$$E_L = \frac{(1-T_C)(C-I)}{R_L} \quad (6)$$

where  $R_L > R_U$  with  $R_L$  positively related to debt (e.g., the cash flow to equity owners has more uncertainty as debt increases). Inserting (6) and (2) into the definition  $V_L = E_L + D$  gives

$$V_L = \frac{(1-T_C)(C-I)}{R_L} + \frac{I}{R_D} \quad (7)$$

where  $R_D = R_F$  only if debt is risk-free debt (as MM assume or as the CAPM suggests when a debt beta is assumed to be zero, which is often the assumption). Regardless, the derivation of the below  $G_L$  formulation is unimpeded if  $R_D$  is endogenously determined by the debt level choice such that  $R_D > R_F$  holds.

The  $G_L$  formulation for an unlevered firm issuing debt can now be derived. After substituting (7) into (4) and noting  $V_U = E_U$ , Appendix A shows

$$G_L = \left[1 - \frac{\alpha R_D}{R_L}\right] D + \left[\frac{R_U}{R_L} - 1\right] E_U \quad (8)$$

where  $\alpha = (1-T_C)$ .

The 1<sup>st</sup> component,  $\left[1 - \frac{\alpha R_D}{R_L}\right] D$ , is always positive if  $D > 0$  since  $\frac{\alpha R_D}{R_L} < 1$ . If  $D = 0$ , then this component is zero. The 2<sup>nd</sup> component,  $\left[\frac{R_U}{R_L} - 1\right] E_U$ , is always negative if  $D > 0$  since  $E_U > 0$  and

$\frac{R_U}{R_L} < 1$ . If  $D = 0$ , then  $R_U = R_L$  and the 2<sup>nd</sup> component (like the 1<sup>st</sup> component) will also be zero

when  $D = 0$  holds. Thus, if  $D = 0$  then (8) implies that  $G_L = 0$ . But if  $D > 0$  then (8) can be either positive or negative depending on which component has the greatest absolute value.

One can note that the 1<sup>st</sup> component is similar to the traditional  $G_L$  formulations except  $\alpha$  is

multiplied by a value less than one (e.g.,  $\frac{R_D}{R_L} < 1$ ) causing the component to be more positive than the traditional  $G_L$  formulations. In looking at the 2<sup>nd</sup> component, we can see that  $G_L$  can be viewed as being related to how much the increase in debt negatively affects outstanding equity through the percentage increase in its discount rate. This relationship is consistent with the intuitive notion that as leverage increases risk (and thus the required rate of return) then the value of the firm should fall accordingly.

### *B. Gain to Leverage Formulation with Personal Taxes and Constant Growth*

When personal taxes are considered, we can show (in a fashion similar to that found in Appendix A and later in Appendix B) that  $G_L$  can still be expressed as (8) if definitions for  $\alpha$ ,  $V_U$ ,  $E_U$ , and  $D$  are modified to incorporate personal tax rates. For example, we still have

$$G_L = \left[ 1 - \frac{\alpha R_D}{R_L} \right] D + \left[ \frac{R_U}{R_L} - 1 \right] E_U$$

for (8) only now we have:  $\alpha = \frac{(1-T_{PE})(1-T_C)}{(1-T_{PD})}$ ;  $V_U = E_U = \frac{(1-T_{PE})(1-T_C)C}{R_U}$ ;  $D = \frac{(1-T_{PD})I}{R_D}$ ; and,  $E_L = \frac{(1-T_{PE})(1-T_C)(C-I)}{R_L}$ . For the 1<sup>st</sup> component to still be positive (when  $D > 0$ ) is now a bit more

complicated. This is because, for  $\frac{\alpha R_D}{R_L} < 1$  to now hold, restrictions must be placed on  $T_C$ ,  $T_{PE}$ , and  $T_{PD}$  (and these restrictions depend on values for  $R_D$  and  $R_L$ ).

Just as the Miller (1977)  $G_L$  formulation given in (3) reduces to the MM formulation given in (1) if  $T_{PE} = T_{PD}$ , so this paper's  $G_L$  formulation given in (8) reduces to (1) if  $R_U = R_L = R_D$  and  $T_{PE} = T_{PD}$ . With definitions for  $\alpha$ ,  $V_U$ ,  $E_L$ , and  $D$  modified to include personal tax rates, equation (8) reduces to the Miller formulation given by (3) if  $R_U = R_L = R_D$ . These reductions reflect the MM derivational procedure that assumes equality of discount rates when denominations (discount rates) are ignored in the factoring process.

Appendix B derives a  $G_L$  equation when both personal taxes and constant growth is considered. Constant growth implies a current dollar change in after-tax cash flows ( $\delta_g$ ), which we define as

$$\delta_g = (1-T_{PE})(1-T_C)(C-I)(\gamma_L^{\text{Target}}) \quad (9)$$

where  $\gamma_L^{\text{Target}}$  is the growth rate when the firm achieves its targeted (and assumedly desired optimal) amount of interest paid. To derive this  $G_L$  equation, definitions for  $V_U$  and  $E_L$  must be modified as follows:  $V_U = E_U = \frac{(1-T_{PE})(1-T_C)C}{R_U - \gamma_U}$  and  $E_L = \frac{(1-T_{PE})(1-T_C)(C-I)}{R_L - \gamma_L}$  where  $\gamma_U$  is the growth rate if the firm is unlevered and  $\gamma_L$  is a growth rate for a given levered situation. For the unlevered growth rate ( $\gamma_U$ ), we have

$$\gamma_U = \frac{\delta_g}{(1-T_{PE})(1-T_C)C} \quad (10)$$

For the levered growth rate ( $\gamma_L$ ), we have

$$\gamma_L = \frac{\delta_g}{(1-T_{PE})(1-T_C)(C-I)} \quad (11)$$

where  $\gamma_L > \gamma_U$  since  $C > (C - I)$ . We can note that *ceteris paribus*  $\gamma_L$  increases as  $I$  increases. Also,  $\gamma_L = \gamma_L^{\text{Target}}$  when the target leverage ratio is achieved.

With  $\gamma_U$  as the growth rate for the unlevered situation and  $\gamma_L$  the growth rate for a given levered situation, Appendix B shows that

$$G_L = \left[ 1 - \frac{\alpha R_D}{R_L - \gamma_L} \right] D + \left[ \frac{R_U - \gamma_U}{R_L - \gamma_L} - 1 \right] E_U \quad (12)$$

where (12) reduces to (8) if there is no growth such that  $\gamma_L = \gamma_U = 0$ . Note that the 1<sup>st</sup> component can become negative if  $\alpha R_D > (R_L - \gamma_L)$  holds, while the 2<sup>nd</sup> component can become positive if  $(R_U - \gamma_U) > (R_L - \gamma_L)$  holds. This can occur for large amounts of debt where  $\gamma_L$  becomes large causing  $(R_L - \gamma_L)$  to become small.

#### IV. Application Using Company Data

This section presents our application, which considers Australian Gas Light Company (AGL Co.), a major retailer of gas and electricity with about three million customers. We attempt to determine  $G_L$  if the suggested target debt-equity choice is reached and simultaneously try to determine if this is the optimal. To achieve this aim we gather needed market data and company data from independent sources that include a firm offering audit, tax, and advisory services (KPMG International) and one offering brokerage services (State One Stockbroking Ltd.). To compute  $G_L$ , we will use equation (12) with all monetary values given in Australian dollars (A\$).

### A. Market and Tax Rate Data for Application

From [http://www.ipart.nsw.gov.au/papers/KPMG\\_February\\_04.pdf](http://www.ipart.nsw.gov.au/papers/KPMG_February_04.pdf), we find a 48 page report on AGL Co. where KPMG estimates values for variables that affect AGL Co.'s valuation. In Table 1, we give KPMG's suggestions (as of February 2004) for market and tax rate data.

| <b>Table 1. Market and Tax Rate Data</b>  |
|---|
| <b>R<sub>R</sub></b> = Real Rate = <b>3.42%</b>   |
| <b>R<sub>INF</sub></b> = Inflation Rate = <b>2.17%</b>  |
| <b>R<sub>F</sub></b> = Risk-Free Rate = $R_R + R_{INF} + (R_R)(R_{INF}) = 3.42\% + 2.17\% + (3.42\%)(2.17\%) = \mathbf{5.6642\%}$                           |
| <b>M<sub>PREM</sub></b> = Market Premium = $(R_M - R_F) = \mathbf{6.00\%}$  |
| <b>T<sub>C</sub></b> = Corporate Tax Rate = <b>30.00%</b>   |
| $\lambda$ = Imputation Tax Credit = <b>40.00%</b>   |
| <b>T<sub>E</sub></b> = Effective Tax Rate = $T_C(1-\lambda) = 30\%(1 - 0.4) = \mathbf{18.00\%}$   |
| <b>T<sub>E</sub></b> = Effective Tax Rate = $\frac{1-T_C}{1-[T_C(1-\lambda)]} = \frac{1-0.3}{1-[0.3(1-0.4)]} = 0.14634$ or <b>14.634%</b>                   |
| <b>Average T<sub>E</sub></b> = $(18.00\% + 14.634\%) = 16.317\% \approx \mathbf{16.32\%}$   |
| <b>Average (1 - T<sub>E</sub>)</b> = $1 - 0.16317 = 0.83683 \approx \mathbf{83.68\%}$   |
| <b>T<sub>PE</sub></b> = Personal Tax Rate on Equity Income = <b>4.77%</b>   |
| <b>T<sub>PD</sub></b> = Personal Tax Rate on Debt Income = <b>20.34%</b>  |
| $\alpha = \frac{(1-T_{PE})(1-T_C)}{(1-T_{PD})} = \frac{(1-0.0477)(1-0.3)}{(1-0.2034)} = 0.83682 \approx \mathbf{83.68\%} \approx \text{Average } (1 - T_E)$ |
| <b>(1 - <math>\alpha</math>)</b> = $(1 - 0.83682) = 0.16318 \approx \mathbf{16.32\%} \approx \text{Average } T_E$   |

KPMG gives no estimates for personal tax rates so we turn to another approach that uses knowledge of imputation credits ( $\lambda$ ). Under the Australian imputation tax system, domestic equity investors receive a taxation credit for dividends paid out of after-tax firm returns. In essence, an imputation tax system offsets the corporate tax advantage of debt in a manner analogous to when equity owners have a lower tax rate than debt owners ( $T_{PE} < T_{PD}$ ). KPMG suggests that  $\lambda = 0.4$  and that  $T_C$  (for equity owners) is effectively reduced to a lower rate ( $T_E$ ). As seen in Table 1, using the average of the computations given by KPMG, we get Average  $T_E = 16.317\% \approx 16.32\%$ . Using this

value to estimate the personal tax rates, we proceed by noting that  $\alpha = (1 - T_E) \approx 1 - 0.1632 \approx 0.8368$  or about 83.68%. This value for  $\alpha$  can be equated with a number of values for  $T_{PE}$  and  $T_{PD}$  (when  $T_C = 30\%$ ) including the two we use in our application:  $T_{PE} = 4.77\%$  and  $T_{PD} = 20.34\%$ . As seen in Table 1, for these two values,  $\alpha \approx 83.68\%$ .

We should point out that after personal tax values for  $E_U$ ,  $D$ ,  $E_L$  and  $V_L$  when  $T_C = 30\%$ ,  $T_{PE} = 4.77\%$ , and  $T_{PD} = 20.34\%$  will differ from those when  $T_C$  (given by  $T_E$ ) = 16.32%,  $T_{PE} = 0$  and  $T_{PD} = 0$ . However, if we look at before personal tax values for  $G_L$ , the reductions in after personal tax values for  $E_U$ ,  $D$ ,  $E_L$  and  $V_L$  caused by using  $T_{PE} = 4.77\%$  and  $T_{PD} = 20.34\%$  will be overcome from the standpoint of getting values before lowered by paying personal taxes.

**Table 2. Beta and Cost of Capital Data**

**Current Book**  $\frac{D}{E} = \frac{\$3,241,500,000}{\$3,153,000,000} \approx 1.0$  implies  $\frac{D}{V} = \mathbf{0.5}$  and  $\frac{E}{V} = \mathbf{0.5}$

$D_{PREM} = \text{Debt Premium} = \beta_D(R_M - R_F) = \beta_D(M_{PREM}) = \mathbf{1.75\%}$

$\beta_D = \text{Debt Beta} = D_{PREM} / M_{PREM} = 1.75\% / 6\% = \mathbf{0.291667}$

$R_D = \text{Cost of Debt} = R_F + \beta_D M_{PREM} = 5.66421\% + 0.291667(6\%) = 7.41421\% \approx \mathbf{7.41\%}$

$\beta_L = \text{Levered Equity Beta} = \mathbf{1.05}$

$R_L = \text{Cost of Levered Equity} = R_F + \beta_L M_{PREM} = 5.6642\% + 1.05(6\%) = 11.9642\% \approx \mathbf{11.96\%}$

$\beta_U = \text{Unlevered Equity Beta} = \frac{\beta_L + [\beta_D (1 - T_C) (\frac{D}{E})]}{1 + [(1 - T_C) (\frac{D}{E})]} = \frac{1.05 + [0.291667(1 - 0.3)(1.0)]}{1 + [(1 - 0.3)(1.0)]} = \mathbf{0.737745}$

$R_U = \text{Cost of Unlevered Equity} = R_F + \beta_U M_{PREM} = 5.664214\% + 0.737745(6\%) = \mathbf{10.0907\%}$

### *B. Beta and Cost of Capital Data for Application*

As seen in Table 2, the annual report for AGL Co.'s fiscal year ending June 2004 indicates its current book  $\frac{D}{E}$  ratio  $\approx 1.0$  where  $D$  is total liabilities and  $E$  is shareholders' equity. The values suggested by KPMG for betas and costs of capital are assumed to correspond with this book debt-equity choice of 1.0. To get AGL Co.'s cost of debt, we begin by noting that KPMG estimates a debt margin of 1.75%, which absent other costs suggests a debt premium ( $D_{PREM}$ ) of 1.75%. As seen in Table 2, this premium implies the debt beta ( $\beta_D$ ) = 0.2917. Using the CAPM, we get the cost of debt

$(R_D) \approx 7.41\%$ , which is the midpoint of KPMG's range estimated at 7.21% to 7.61%. KPMG suggests AGL Co.'s levered beta  $(\beta_L) = 1.05$ . Using the CAPM, this value for  $\beta_L$  implies its cost of levered equity  $(R_L) \approx 11.96\%$ .

KPMG indicates, regardless of any formula chosen,  $B_D$  should not be assumed zero when computing the unlevered beta  $(\beta_U)$ . Given  $T_C = 30\%$ , the current book leverage ratios ( $\frac{D}{E} = 1.0$ ,  $\frac{D}{V} = 0.5$ , and  $\frac{E}{V} = 0.5$ ),  $\beta_L = 1.05$ , and  $\beta_D = 0.29$ , Table 2 uses the formula given by Hamada (1972) to get  $\beta_U = 0.7377$ . Using the CAPM, this value for  $\beta_U$  implies the cost of unlevered equity  $(R_U) = 10.0907\%$ .

Given the data in Table 2, we can try to estimate betas (and thus  $R_D$ 's and  $R_L$ 's) for various debt level choices. For simplicity, we consider only nine  $\frac{D}{V}$  choices with each choice given in book values (before market adjustments caused by a positive  $G_L$  are made). We determine each choice based on the number of shares retired. This is accomplished by noting AGL Co. has outstanding levered shares ( $N_L$ ) of 456,000,000. Given  $N_L$  and current book  $\frac{E}{V} = 0.5$ , we can estimate the number of shares if the firm was unlevered ( $N_U$ ). As will be seen in Table 3, we have

$$N_U = \frac{N_L}{\text{Current Book } \frac{E}{V}} = \frac{456,000,000}{0.5} = 912,000,000.$$

From this  $N_U$  value, we can get the number of shares retired ( $S_R$ ) for each debt level choice. For example, if  $\frac{E}{V} = 0.9$ , then  $\frac{D}{V} = 0.1$  and AGL Co. would retire  $S_R = (\frac{D}{V})(N_U) = 0.1(912,000,000) = 91,200,000$  shares. Similarly, we can compute  $S_R$  for any debt level choice. For each choice faced by an unlevered firm, the value of the debt issued ( $D$ ) should *ceteris paribus* equal the dollar amount of the retired shares where  $D = P_U(S_R)$  with  $P_U$  the unlevered share price (which can be viewed as the current market price minus any gains to leverage).

Based on our nine  $\frac{D}{V}$  choices from 0.1 to 0.9, we can attempt to construct nine corresponding  $\beta_D$ 's and  $\beta_L$ 's from which we can compute  $R_D$ 's and  $R_L$ 's needed in our  $G_L$  formulation. We begin by estimating debt betas ( $\beta_D$ 's). The estimation process is based on the observation that we have two  $\beta_D$ 's endpoints and a  $\beta_D$  interior point. This is seen below.

For the 1<sup>st</sup> endpoint when  $\frac{D}{V} = 0$ , we have  $\beta_D = 0$ .

For the interior point when  $\frac{D}{V} = 0.5$ , we have (from Table 2)  $\beta_D = 0.2977$ .

For the 2<sup>nd</sup> endpoint when  $\frac{D}{V} \rightarrow 1.0$ , we have  $\beta_D \rightarrow \beta_U = 0.7377$ .

Concerning the latter, we see that as a firm approaches an all-debt situation with all shares retired, it must legally revert to an all-equity firm and the unlevered beta of 0.7377.

Using linear interpolation, we can estimate the  $\beta_D$ 's for each book  $\frac{D}{V}$  choice from 0.1 to 0.9. We can then use the CAPM to get each corresponding  $R_D$ . We show the  $\beta_D$  and  $R_D$  values below.

| <b>Debt Betas &amp; Costs of Debt for Book <math>\frac{D}{V}</math> Choices from 0.1 to 0.9</b> |       |       |       |       |       |       |       |       |       |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|   | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9   |
| $\beta_D$   | 0.058 | 0.117 | 0.175 | 0.233 | 0.292 | 0.381 | 0.470 | 0.559 | 0.649 |
| $R_D$   | 6.01% | 6.36% | 6.71% | 7.06% | 7.41% | 7.94% | 8.48% | 9.02% | 9.56% |

For  $\beta_L$ 's, we have only one endpoint ( $\frac{D}{V} = 0$ ,  $\beta_L = \beta_U = 0.7377$ ) and an interior point ( $\frac{D}{V} = 0.5$ ,  $\beta_L = 1.05$ ), ruling out linear interpolation for all choices. Given  $\beta_D$ 's and  $\beta_U$ , we use Hamada (1972) to estimate  $\beta_L$ 's. However, estimates using this equation break down as we approach high debt levels because the  $\beta_L$ 's values generate the same small linear increase of 0.06245 for each successive book  $\frac{D}{V}$  choice from 0.6 to 0.9. Because of its linear relationship that treats the latter incremental increases in debt as equally risky, the Hamada equation clearly cannot accommodate any expected rapidly increasing levels of financial risk as we reach extreme debt levels. Thus, for the last two choices, our application uses  $\beta_L$ 's of 1.42 and 2.00 instead of the linear values given by Hamada (1.23735 and 1.29980).  $\beta_L$ 's of 1.42 and 2.00 represent an increase of about 20% and 40% over respective previous  $\beta_L$ 's, and are deemed a more acceptable attempt to capture the increasing levels of financial risk. The decision to start increasing  $\beta_L$ 's for the 8<sup>th</sup> and 9<sup>th</sup> debt level choice is consistent with the target leverage ratio that is set for AGL Co. so as to avoid undue financial distress. It remains for future research to explore if there can be found a formula for  $\beta_L$  supporting the observation that firms do not strive for extreme high levels of debt.

Below we display the  $\beta_L$ 's and  $R_L$ 's for the book  $\frac{D}{V}$  choices from 0.1 to 0.9 with the CAPM used to compute  $R_L$ 's.

| <b>Equity Betas &amp; Costs of Equity for Book <math>\frac{D}{V}</math> Choices from 0.1 to 0.9</b> |        |        |        |        |        |        |        |        |        |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|   | 0.1    | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    |
| $\beta_L$   | 0.800  | 0.863  | 0.925  | 0.988  | 1.050  | 1.112  | 1.174  | 1.420  | 2.000  |
| $R_L$   | 10.47% | 10.84% | 11.21% | 11.59% | 11.96% | 12.34% | 12.71% | 14.18% | 17.66% |

### *C. Data Related to Unlevered Situation and Market Target $\frac{D}{E}$ for Application*

Given  $R_L$ 's and  $R_D$ 's for the nine debt-equity choices, we can get  $E_U$  and  $E_L$  by estimating values for  $C$ ,  $\gamma_L^{\text{Target}}$ ,  $P_U$ ,  $I$ ,  $\delta_g$ , and  $\gamma_U$ . We estimate the perpetual before-tax cash flow for unlevered equity owners (i.e., we estimate  $C$ ) from <http://www.stateone.com.au/documents/research/agl.pdf>, which gives earnings before interest and taxes plus depreciation / amortization (EBITDA). For the years from 2003 to 2006, the average EBITDA suggests that  $C = \$905.2$  million.

Next, we estimate the growth rate for an unlevered situation ( $\gamma_U$ ). We begin by trying to determine current dollar change in after-tax cash flows ( $\delta_g$ ) as given by (9) where  $\delta_g = (1-T_{PE})(1-T_C)(C-I)(\gamma_L)$  with  $\gamma_L$  defined as the levered growth rate when the firm achieves its targeted amount of interest paid (e.g.,  $\gamma_L = \gamma_L^{\text{Target}}$ ). Noting that KPMG indicates that AGL Co.'s target  $\frac{D}{E}$  ratio is 1.5 (which we take as the market ratio since KPMG uses the word "market" when describing the weights from this choice), we proceed to estimate the future growth rate for after-tax cash flows given this target. We settle on  $\gamma_L^{\text{Target}} = 5.4\%$ . This estimate is consistent with data suggested by State One and AGL Co. reports. For example, State One (December 2004) estimates that the net profit after tax will change from \$320.8 million for June 2003 to \$376.4 million for June 2006. The implied growth rate in after-tax cash flows =  $\gamma_L = \left(\frac{\$376.4}{\$320.8}\right)^{\frac{1}{3}} - 1 \approx 5.5\%$ . A similar value is found ( $\gamma_L \approx 5.3\%$ ) if we take the average of the growth in dividends from June 2003 to June 2006 and the standard formula where the growth rate equals retention rate times required rate of return. To proceed with estimating  $\delta_g$ , we next need to compute the interest paid ( $I$ ) when  $\gamma_L = 5.4\%$ . As will be seen later in Table 5, it is for the 7<sup>th</sup> debt level choice when 70% of unlevered shares are retired (book choice of  $\frac{D}{V} = 0.7$ ) that we attain the market target  $\frac{D}{E} \approx 1.5$  ( $\frac{D}{E} \approx 2.33$  from a share standpoint). As we will show later,  $I = \$493,093,903$  for this 7<sup>th</sup> choice. Using equation (9), Table 3

shows that  $\delta_g = \$14,834,558$ . This estimate is reasonably close to State One's average increase in NPAT for 2003–2006 of about \$15.7 million if adjusted for personal taxes. Given this value, we can now use (10) to solve for  $\gamma_U$  where we obtain  $\gamma_U \approx 2.46\%$ . Given  $\gamma_U$ , we can proceed to compute the unlevered equity value ( $E_U$ ). Table 3 shows that  $E_U = \$7,906,124,561$ . On a before personal tax basis,  $E_U = \$8,302,136,471$ . Dividing this by the number of unlevered shares ( $N_U$ ), we compute the stock price for the unlevered situation ( $P_U$ ) and get about \$9.10 as shown in Table 3.

| <b>Table 3. Data Related to Unlevered Situation and Target Market <math>\frac{D}{E}</math></b>  |                        |
|---|------------------------|
| $N_L$ = Number of Shares when Levered with Current Book $\frac{D}{E}$ is 1.0 =  | <b>456,000,000</b>     |
| $N_U$ = Number of Shares when Unlevered = $\frac{N_L}{\text{Current Book } \frac{E}{V}}$ = $\frac{456,000,000}{0.5}$ =                                | <b>912,000,000</b>     |
| $C$ = Estimated by Average EBITDA (2003–2006) =   | <b>\$905,200,000</b>   |
| <b>Target Market <math>\frac{D}{E}</math></b> = Suggested Market Target Debt-Equity Choice =  | <b>1.5</b>             |
| $\gamma_L = \gamma_L^{\text{Target}}$ = Estimated Growth Rate in After-Tax Cash Flows for Target Market $\frac{D}{E}$ =                               | <b>5.4%</b>            |
| $I$ = Interest Paid for Targeted Levered Situation =  | <b>\$493,093,903</b>   |
| $\delta_g = (1 - T_{PE})(1 - T_C)(C - I)\gamma_L^{\text{Target}}$ = $(1 - 0.0477)(1 - 0.3)(\$905,200,000 - \$493,093,903)0.054$ =                     | <b>\$14,834,558</b>    |
| $\gamma_U = \frac{\delta_g}{(1 - T_{PE})(1 - T_C)C}$ = $\frac{\$14,834,558.46}{(1 - 0.0477)(1 - 0.3)\$905,200,000}$ = 2.458432% $\approx$             | <b>2.46%</b>           |
| $E_U$ (After Personal Taxes) = $\frac{(1 - T_{PE})(1 - T_C)C}{R_U - \gamma_U}$ = $\frac{(1 - 0.0477)(1 - 0.3)\$905,200,000}{0.100907 - 0.02458432}$ = | <b>\$7,906,124,561</b> |
| $E_U$ (Before Personal Taxes) = $\frac{\$7,906,124,561}{(1 - T_{PE})}$ = $\frac{\$7,906,124,561}{(1 - 0.0477)}$ =                                     | <b>\$8,302,136,471</b> |
| $P_U$ (Per Share Unlevered Market Price) = $\frac{E_U}{N_U}$ = $\frac{\$8,302,136,471}{912,000,000}$ =  | <b>\$9.10322</b>       |

#### *D. Company Data Related to Market Debt-Equity Target for Application*

Given  $\delta_g$  and  $P_U$ , we can now compute the interest paid ( $I$ ) for each debt level choice given that  $I = R_D(D)$  where as described earlier  $D = P_U(S_R)$ . Because we have unlevered the firm where  $N_U =$

912,000,000 and  $P_U = \$9.10322$ , we can view each debt level choice for our unlevered firm as  $D = P_U(S_R)$  where  $S_R$  is the number of shares being retired for that choice. Since  $S_R = (\frac{D}{V})(N_U)$ , we have  $D = P_U(\frac{D}{V})(N_U)$ . Inserting this expression for  $D$  into  $I = R_D(D)$ , we get  $I = R_D(P_U)(\frac{D}{V})(N_U)$ . Although the details are omitted, we can note that from this relationship, we solve for the earlier value for  $I = \$493,093,903$  for the 7<sup>th</sup> debt level choice. This is because we can create a quadratic equation where  $I$  is a function of  $T_C$ ,  $T_{PE}$ ,  $C$ ,  $R_D$ ,  $R_U$ ,  $\gamma_L^{\text{Target}}$ ,  $N_U$ , and  $S_R$ , all of which are known.

| <b>Table 4. Company Data Related to Market Debt-Equity Target</b><br>(Values for Market Debt-Equity Choice = 1.5 which is Book Debt-Firm Value Choice = 0.7) |  |
|--|--|
| <b>R</b>   | Unlevered Shares Exchanged = $\frac{D}{V} N_U = 0.7(912,000,000) = \mathbf{638,400,000}$ shares  |
| <b>Book <math>\frac{D}{V}</math></b>   | Book Value Leverage Choice Given by Shares Retired = $\frac{R}{N_U} = \frac{638,400,000}{912,000,000} = \mathbf{0.7}$  |
| <b>D (Before Personal Tax)</b>   | Amount of $E_U$ Retired = $P_U(R) = \$9.10322(638,400,000) = \mathbf{\$5,811,495,529}$   |
| <b>I (Interest Paid)</b>   | $R_D D = 0.084848(\$5,811,495,529) = \mathbf{\$493,093,903}$   |
| <b><math>E_L</math></b>  | $E_L = \frac{(1 - T_{PE})(1 - T_C)(C - I)}{R_L - \gamma_L} = \frac{(1 - 0.0477)(1 - 0.3)(\$905,200,000 - \$493,093,903)}{0.12713626 - 0.054} = \mathbf{\$3,756,195,004}$   |
| <b>D</b>   | $D = \frac{(1 - T_{PD})I}{R_D} = \frac{(1 - 0.2034)\$493,093,903}{0.08484802} = \mathbf{\$4,629,437,339}$  |
| <b>Target Market <math>\frac{D}{E}</math></b>  | (On Before Personal Tax Basis) = $\frac{\$4,629,437,339 / (1 - 0.2034)}{\$3,756,195,004 / (1 - 0.0477)} = 1.473 \approx \mathbf{1.5}$  |
| <b><math>G_L</math> using (12)</b>   | $G_L \text{ using (12)} = \left[ 1 - \frac{\alpha R_D}{R_L - \gamma_L} \right] D + \left[ \frac{R_U - \gamma_U}{R_L - \gamma_L} - 1 \right] E_U = \$135,068,376 + \$344,439,406 = \mathbf{\$479,507,782}$          |
| <b>1<sup>st</sup> Component</b>  | $1^{\text{st}} \text{ Component} = \left[ 1 - \frac{\alpha R_D}{R_L - \gamma_L} \right] D = \left[ 1 - \frac{(0.836819)(0.08484802)}{0.12713626 - 0.054} \right] \$4,629,437,339 = \mathbf{\$135,068,376}$         |
| <b>2<sup>nd</sup> Component</b>  | $2^{\text{nd}} \text{ Component} = \left[ \frac{R_U - \gamma_U}{R_L - \gamma_L} - 1 \right] E_U = \left[ \frac{0.10090685 - 0.024584323}{0.12713626 - 0.054} - 1 \right] \$7,906,124,561 = \mathbf{\$344,439,406}$ |
| <b><math>V_L</math></b>  | $V_L = E_L + D = \$3,756,195,004 + \$4,629,437,339 = \mathbf{\$8,385,632,343}$   |
| <b><math>G_L</math> using (4)</b>  | $G_L \text{ using (4)} = V_L - V_U = \$8,385,632,343 - \$7,906,124,561 = \mathbf{\$479,507,782}$   |

With our firm unlevered, we can now illustrate the computation I for any debt level choice from this unlevered condition, which is the condition assumed to apply when using (12). Consider the 1<sup>st</sup> choice where book  $\frac{D}{V} = 0.1$  and  $R_D = 6.0142\%$ . We have:  $I = R_D (P_U)(\frac{D}{V})(N_U) = 0.06014214(\$9.10322)(0.1)(912,000,000) = \$49,930,825$ . Similarly, we can compute I for all choices. For the 7<sup>th</sup> choice, we can verify that  $I = \$493,093,903$  for book  $\frac{D}{V} = 0.7$ . We have:  $I = R_D (P_U)(\frac{D}{V})(N_U) = 0.084848(\$9.10322)(0.7)(912,000,000) = \$493,093,903$ . As seen in Table 4, this interest payment corresponds very closely to the target market debt-equity choice of 1.5.

After computing I for each debt level choice, we can use (11) and compute  $\gamma_L$  for each choice. For example, for the 1<sup>st</sup> debt level choice, we get

$$\gamma_L = \frac{\delta_g}{(1 - T_{PE})(1 - T_C)(C - I)} = \frac{\$14,834,558.46}{(1 - 0.0477)(1 - 0.3)(\$905,200,000 - \$49,930,825)} = 2.601956\%.$$

We obtain the following  $\gamma_L$  values for the nine debt level choices: 2.602%, 2.783%, 3.016%, 3.318%, 3.725%, 4.370%, 5.400%, 7.270%, and 11.637%. The growth rates begin increasing much more rapidly as  $I \rightarrow C$  causing the 1<sup>st</sup> and 2<sup>nd</sup> components of (12) to eventually flip signs.

Given  $\gamma_L$  for each debt level choice, we can use (12) or (4) to get  $G_L$  for each corresponding debt level choice. We do this in Table 4 for the 7<sup>th</sup> debt level choice and show that  $G_L = \$479,507,782$  on an after personal tax basis. The table also shows that this choice corresponds with a target market  $\frac{D}{E} \approx 1.5$  when computed on a before personal tax basis.

## V. Results for All Nine Debt Level Choices for the Application

This section summarizes the results of our application in table form. We give gain to leverage ( $G_L$ ) results for the unlevered application for AGL Co. for all nine debt level choices. The application assumes the previously mentioned data including the betas needed to compute the costs of capital.

Conditions of our application are formally stated below so as to include values for key variables.

- (a) debt is risky with  $R_D > R_F = 5.6642\%$ , and  $R_D$  is positively related to debt.
- (b) tax rates are relevant with  $T_{PE} = 4.77\%$ ,  $T_{PD} = 20.34\%$ , and  $T_C = 30\%$ ;
- (c) uncertain perpetual before-tax cash flows to unlevered equity:  $C = \$905,200,000$ ;
- (d) constant growth rate when target market  $\frac{D}{E}$  approximated:  $\gamma_L^{\text{target}} = 5.4\%$  with current dollar change ( $\delta_g$ ) = \$14,834,558; and,
- (e) an unlevered firm with risky equity faces a finite set of perpetual debt-for-equity choices with  $R_L > R_U = 10.0907\%$ .

**Table 5. Application of Gain to Leverage Formulation for a Real World Firm Assuming Risky Debt, Personal Taxes, and Constant Growth Rate**

| <b>Panel A. On After Personal Tax Basis with Currency in Billions of Australian Dollars</b>  |               |               |   |   |               |              |              |              |                                |
|--|---------------|---------------|---|---|---------------|--------------|--------------|--------------|--------------------------------|
| <b>Book</b><br>$\frac{D}{V}$   | $R_D$         | $R_L$         | <b>1<sup>st</sup></b><br><b>Component</b> | <b>2<sup>nd</sup></b><br><b>Component</b> | $G_L$         | <b>D</b>     | $E_L$        | $V_L$        | <b>Market</b><br>$\frac{D}{E}$ |
| 0.1  | 0.0601        | 0.1047        | 0.2381                                    | -0.2324                                   | 0.0056        | 0.661        | 7.250        | 7.912        | 0.091                          |
| 0.2  | 0.0636        | 0.1084        | 0.4484                                    | -0.4165                                   | 0.0318        | 1.323        | 6.615        | 7.938        | 0.200                          |
| 0.3  | 0.0671        | 0.1121        | 0.6245                                    | -0.5467                                   | 0.0777        | 1.984        | 6.000        | 7.984        | 0.331                          |
| 0.4  | 0.0706        | 0.1159        | 0.7547                                    | -0.6106                                   | 0.1440        | 2.645        | 5.405        | 8.050        | 0.489                          |
| <b>0.5</b>   | <b>0.0741</b> | <b>0.1196</b> | <b>0.8167</b>                             | <b>-0.5825</b>                            | <b>0.2341</b> | <b>3.307</b> | <b>4.834</b> | <b>8.140</b> | <b>0.684</b>                   |
| 0.6  | 0.0795        | 0.1234        | 0.6555                                    | -0.3338                                   | 0.3217        | 3.968        | 4.260        | 8.228        | 0.932                          |
| <b>0.7</b>   | <b>0.0848</b> | <b>0.1271</b> | <b>0.1351</b>                             | <b>0.3444</b>                             | <b>0.4795</b> | <b>4.629</b> | <b>3.756</b> | <b>8.386</b> | <b>1.232</b>                   |
| 0.8  | 0.0902        | 0.1418        | -0.4850                                   | 0.8208                                    | 0.3358        | 5.291        | 2.951        | 8.246        | 1.793                          |
| 0.9  | 0.0956        | 0.1766        | -1.9447                                   | 2.1058                                    | 0.1611        | 5.952        | 2.115        | 8.067        | 2.814                          |
| <b>Panel B. On Before Personal Tax Basis with Currency in Billions of Australian Dollars</b> |               |               |   |   |               |              |              |              |                                |
| <b>Book</b><br>$\frac{D}{V}$   | $R_D$         | $R_L$         | <b>1<sup>st</sup></b><br><b>Component</b> | <b>2<sup>nd</sup></b><br><b>Component</b> | $G_L$         | <b>D</b>     | $E_L$        | $V_L$        | <b>Market</b><br>$\frac{D}{E}$ |
| 0.1  | 0.0601        | 0.1047        | 0.3857                                    | -0.2441                                   | 0.1417        | 0.830        | 7.614        | 8.444        | 0.109                          |
| 0.2  | 0.0636        | 0.1084        | 0.7422                                    | -0.4374                                   | 0.3049        | 1.660        | 6.947        | 8.607        | 0.239                          |
| 0.3  | 0.0671        | 0.1121        | 1.0630                                    | -0.5741                                   | 0.4888        | 2.491        | 6.300        | 8.791        | 0.395                          |
| 0.4  | 0.0706        | 0.1159        | 1.3354                                    | -0.6412                                   | 0.6942        | 3.321        | 5.676        | 8.996        | 0.585                          |
| <b>0.5</b>   | <b>0.0741</b> | <b>0.1196</b> | <b>1.5363</b>                             | <b>-0.6117</b>                            | <b>0.9246</b> | <b>4.151</b> | <b>5.076</b> | <b>9.227</b> | <b>0.818</b>                   |
| 0.6  | 0.0795        | 0.1234        | 1.5028                                    | -0.3505                                   | 1.1523        | 4.981        | 4.473        | 9.454        | 1.114                          |
| <b>0.7</b>   | <b>0.0848</b> | <b>0.1271</b> | <b>1.0920</b>                             | <b>0.36170</b>                            | <b>1.4537</b> | <b>5.811</b> | <b>3.944</b> | <b>9.756</b> | <b>1.473</b>                   |
| 0.8  | 0.0902        | 0.1418        | 0.5767                                    | 0.8619                                    | 1.4386        | 6.642        | 3.099        | 9.741        | 2.143                          |
| 0.9  | 0.0956        | 0.1766        | -0.8205                                   | 2.2113                                    | 1.3908        | 7.472        | 2.221        | 9.693        | 3.364                          |

Table 5 reports results for ten key variables when financial managers face nine debt choices given in the nine rows. Panel A reports results (where applicable) on an after personal tax basis while Panel B focuses on before personal tax results. As discussed previously, the before personal tax results are important because they can minimize inaccuracies in the after personal tax results that might result if personal tax rates are closer in value than what we use.

Each panel has two bold-faced rows. The 1<sup>st</sup> bold-faced row is for the current situation where book  $\frac{D}{V} = 0.5$ , while the 2<sup>nd</sup> bold-faced row is for book  $\frac{D}{V} = 0.7$ , which is where  $G_L$  is maximized for both panels. As seen in the last column of Panel B, it is also the row which is nearest the market target  $\frac{D}{E}$  of 1.5. For this row, we get  $G_L = \$1.4537$  billion on a before personal tax basis (which is what the market sees). For this row, dividing  $E_L$  by the number of outstanding shares ( $N_L$ ), we get a share price  $\approx \$14.42$ . For example, with  $\frac{D}{V} = 0.7$  (or  $\frac{E}{V} = 0.3$ ), we have  $N_L = \frac{E}{V} (N_U) = 0.3(912,000,000) = 273,600,000$  shares giving the market share price as:

$$P_{\text{Before Personal Tax}} = \frac{E_L}{N_L} = \frac{\$3,944,340,023}{273,600,000 \text{ shares}} = \$14.4164 \text{ per share.}$$

This is less than the average market price at the time of this writing, which has averaged \$13.83 for January 2005. Thus, \$14.42 can be considered a prediction of the future price (absent effects beyond those stemming from the increased debt) if the market target is achieved.

The prediction for the stock price at the time we begin estimating values for our variables (February 2004) can be computed for the 1<sup>st</sup> bold-faced row where  $N_L = 456,000,000$  shares. We have:

$$P_{\text{Before Personal Tax}} = \frac{E_L}{N_L} = \frac{\$5,075,687,792}{456,000,000 \text{ shares}} = \$11.1309 \text{ per share.}$$

This price is consistent with both the average price of \$11.06 for AGL Co. for February 2004 and also for the average price of \$11.29 for the year of the 2003 annual report (7/1/03 to 6/30/04).

$G_L$  on a before personal tax basis in Panel B is greater than the after personal tax basis in Panel A given that personal taxes are subtracted from  $G_L$  in Panel A. The difference is sizeable as seen when comparing the maximum  $G_L$  value of \$0.4795 billion in Panel A with the corresponding maximum value of \$1.4537 billion in Panel B. In looking at Panel B, we can also point out, that due

to the increase in equity that accrues from  $G_L$ , the book  $\frac{D}{E}$  of 2.333 (from a share standpoint) is reduced to a market  $\frac{D}{E}$  of 1.473. This is near the target of 1.5.

While the maximum  $G_L$  is achieved in both panels with the 7<sup>th</sup> debt level choice, it does not necessarily follow that both panels will agree. It may be even more likely that the after personal tax  $G_L$  can be achieved for a different debt level choice than the before personal  $G_L$ . If decisions are actually made on what is best for investors, the firm would arguably choose the debt level where  $G_L$  is maximized on an after personal tax basis. However, absent perfect knowledge about personal tax rates and given our restriction to nine debt level choices (where this restriction tends to underestimate the maximum  $G_L$  and the optimal  $\frac{D}{E}$ ), it appears that the firm's maximum  $G_L$  on a before personal tax basis will occur within a  $\frac{D}{E}$  range of about 1.4 to 2.0. This range is consistent with values reported by <http://www.bizstats.com/currentratios.htm>. For example, BizStats give a debt-to-equity ratio of 1.79 for U.S.A. gas production and distribution utilities.

Although both  $G_L$  components experience change in signs as more debt is added, this is not necessarily always the case. Absent a large value for  $\delta_g$  that leads to large values for  $\gamma_L$  for high debt levels, we would expect the 1<sup>st</sup> component to always be positive and the 2<sup>nd</sup> component to always be negative.

## **VI. Limitations of Application**

In this section, we call attention to four limitations facing our application. In general, such limitations are found in all models that rely on accurate estimates of values for its variables.

First, personal tax rates were not directly known. This limitation was ameliorated through use of an effective tax rate and analysis of before personal tax values.

Second, most firms are levered. Thus, to apply our  $G_L$  formulation, we have to unlever our firm in an attempt to estimate the number of shares outstanding ( $N_U$ ) if it had no debt. From here we determine book debt-to-equity choices. Given these choices and the unlevered price ( $P_U$ ), we can determine how much debt will be issued for each choice. The application depends on the accuracy of estimating  $N_U$  and  $P_U$  and is limited if these estimates lack sufficient accuracy.

Third, we encountered a roadblock when computing betas. For example, we had to interpolate from endpoints and a midpoint to get reasonable  $\beta_D$ 's for each debt level choice. From there we

proceeded to get  $\beta_U$  and then obtain  $\beta_L$ 's for the nine debt level choices by using standard formulas. However, unless adjusted upward, the  $\beta_L$  computations for higher debt levels would suggest that firms aim for extremely high leverage targets, which we do not find in the real world. This limitation (in getting sufficient estimates for at least some betas) caused us to make intuitive ad hoc assignments for levered equity betas at higher levels of debt. Future research needs to explore other ways of estimating betas and costs of capital such as suggested by researchers who offer alternatives methods (Fama and French, 1997; Lally, 2004).

Finally, the application had to estimate a current dollar level of growth ( $\delta_g$ ) based on a chosen growth rate at the target debt-equity choice. Using Excel, we were able to solve for  $\delta_g$  and the interest paid (I) at the market target debt-equity choice based on values for other variables.

## **VII. Summary Statements**

This paper derives  $G_L$  formulations based on definitions for unlevered and levered firm values. Such formulations include discount rates for unlevered equity, levered equity, and debt. The inclusion of these rates makes it possible for  $G_L$  values to eventually decrease with increasing debt levels. Three  $G_L$  formulations for an unlevered situation are offered to aid managers (when making the debt-equity choice) and educators (when explaining the ramifications of the debt-equity choice).

The application using market data and company data for AGL Co. showed how managers can use the  $G_L$  formulation with personal taxes and constant growth to understand how the debt-equity choice can influence firm value. While this paper's model (like any model) relies on accurate estimates of values for variables, the model's optimal  $G_L$  was able to conform to the recommended market target  $\frac{D}{E}$  of 1.5 by assuming escalating values for levered equity's beta at higher levels of debt.

This study is important for several reasons. First, prior research offers formulations that are difficult for practitioners in that they do not fully address the role of discount rates, and tend to be unrealistic by including variables that are virtually immeasurable in themselves (e.g., direct and indirect bankruptcy costs and agency costs). As such, financial managers are hard pressed to find utility in their application. To the extent changes in discount rates are easier to estimate, this paper's  $G_L$  formulations offer more practical potential.

Second, the practical application in this paper suggests a wealth maximizing debt-equity choice

where the actual choice can depend on taxes and growth rates in addition to changes in discount rates. The application produces results that re-enforce and strengthen general conclusions of prior empirical and theoretical research in regard to the belief that increasing levels of debt can cause  $G_L$  to begin falling. The increase in  $G_L$ , followed by a decrease, can be explained through the interaction of many factors (such as taxes, bankruptcy costs, and agency effects) that together determine a firm's optimal debt-equity choice.

In conclusion, the  $G_L$  formulations found in this paper reaffirm, synthesize, and extend prior  $G_L$  formulations, while opening up a fresh vista from which to view the debt-equity choice faced by managers. This vista offers a practical vantage point in that capital structure decision-making can be based on formulations that include variables heretofore not fully utilized.

### **Appendix A: $G_L$ for Unlevered Firm with No Personal Taxes**

*Proof of equation (8):* For the situation of an unlevered firm when only corporate taxes are considered, substituting (7) into (4) and noting  $V_U = E_U$  gives

$$G_L = \frac{(1 - T_C)(C - I)}{R_L} + \frac{I}{R_D} - E_U.$$

Multiplying out the 1<sup>st</sup> component, noting  $\frac{I}{R_D} = D$ , and rearranging:

$$G_L = D - \frac{(1 - T_C)I}{R_L} - E_U + \frac{(1 - T_C)C}{R_L}.$$

Multiplying the 2<sup>nd</sup> component by  $\frac{R_D}{R_D}$  gives  $-(1 - T_C)\left(\frac{R_D}{R_L}\right)\frac{I}{R_D}$ , which is  $-(1 - T_C)\left(\frac{R_D}{R_L}\right)D$ , and factoring out D:

$$G_L = \left[1 - (1 - T_C)\left(\frac{R_D}{R_L}\right)\right]D - E_U + \frac{(1 - T_C)C}{R_L}.$$

Multiplying the last component by  $\frac{R_U}{R_U}$  gives  $\left(\frac{R_U}{R_L}\right)\frac{(1 - T_C)C}{R_U}$ , which is  $\left(\frac{R_U}{R_L}\right)E_U$ , and factoring out  $E_U$ :

$$G_L = \left[1 - (1 - T_C)\left(\frac{R_D}{R_L}\right)\right]D - \left[1 - \frac{R_U}{R_L}\right]E_U.$$

Setting  $\alpha = (1 - T_C)$  and noting  $-\left[1 - \frac{R_U}{R_L}\right]E_U = +\left[\frac{R_U}{R_L} - 1\right]E_U$  gives

$$G_L = \left[1 - \frac{\alpha R_D}{R_L}\right]D + \left[\frac{R_U}{R_L} - 1\right]E_U. \quad (8)$$

*Q.E.D.*

## Appendix B: $G_L$ for Unlevered Firm with Personal Taxes and Constant Growth

*Proof of equation (12):* Assume constant growth such that  $\gamma_L > \gamma_U > 0$  and personal taxes such

that  $V_U = E_U = \frac{(1 - T_{PE})(1 - T_C)C}{R_U - \gamma_U}$  and  $E_L = \frac{(1 - T_{PE})(1 - T_C)(C - I)}{R_L - \gamma_L}$ . Substituting  $V_L = E_L + D =$

$\frac{(1 - T_{PE})(1 - T_C)(C - I)}{R_L - \gamma_L} + D$  into (4) and noting  $V_U = E_U$  gives:

$$G_L = \frac{(1 - T_{PE})(1 - T_C)(C - I)}{R_L - \gamma_L} + D - E_U.$$

Multiplying out the 1<sup>st</sup> component and rearranging:

$$G_L = D - \frac{(1 - T_{PE})(1 - T_C)I}{R_L - \gamma_L} - E_U + \frac{(1 - T_{PE})(1 - T_C)C}{R_L - \gamma_L}.$$

Multiplying the 2<sup>nd</sup> component by  $\frac{(1 - T_{PD})R_D}{(1 - T_{PD})R_D}$  gives  $-\left(\frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}\right)\left(\frac{R_D}{R_L - \gamma_L}\right)\frac{(1 - T_{PD})I}{R_D}$ ,

which is  $-\left(\frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}\right)\left(\frac{R_D}{R_L - \gamma_L}\right)D$ , and factoring out D:

$$G_L = \left[1 - \left(\frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}\right)\left(\frac{R_D}{R_L - \gamma_L}\right)\right]D - E_U + \frac{(1 - T_{PE})(1 - T_C)C}{R_L - \gamma_L}.$$

Multiplying the last component by  $\frac{R_U - \gamma_U}{R_U - \gamma_U} = 1$  gives  $\left(\frac{R_U - \gamma_U}{R_L - \gamma_L}\right)\frac{(1 - T_{PE})(1 - T_C)C}{R_U - \gamma_U}$ , which is

$\left(\frac{R_U - \gamma_U}{R_L - \gamma_L}\right)E_U$ , and factoring out  $E_U$ :

$$G_L = \left[1 - \left(\frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}\right)\left(\frac{R_D}{R_L - \gamma_L}\right)\right]D - \left[1 - \frac{R_U - \gamma_U}{R_L - \gamma_L}\right]E_U.$$

Setting  $\alpha = \left(\frac{(1 - T_{PE})(1 - T_C)}{(1 - T_{PD})}\right)$  and noting  $-\left[1 - \frac{R_U - \gamma_U}{R_L - \gamma_L}\right]E_U = +\left[\frac{R_U - \gamma_U}{R_L - \gamma_L} - 1\right]E_U$  gives

$$G_L = \left[1 - \frac{\alpha R_D}{R_L - \gamma_L}\right]D + \left[\frac{R_U - \gamma_U}{R_L - \gamma_L} - 1\right]E_U. \quad (12)$$

*Q.E.D.*

## References

- Agrawal, Anup and Nandu Nagarajan, 1990, "Corporate Capital Structure, Agency Costs, and Ownership Control: The Case of All-Equity Firms," *Journal of Finance*, 45 (3), 1325-1331.
- Altman, Edward, 1984, "A Further Empirical Investigation of the Bankruptcy Cost Question," *Journal of Finance*, 39 (September), 1067-1087.
- Baxter, Nevins, 1967, "Leverage Risk of Ruin and the Cost of Capital." *Journal of Finance* 22 (September), 395-403.
- BizStats.com, [December 2004] report on Current Ratios and Other Balance Sheet Ratios. Retrieved Dec. 7, 2004. <http://www.bizstats.com/currentratios.htm>
- Copeland, T. and Won Lee, 1991, "Exchange Offers and Stock Swaps—New Evidence," *Financial Management* 20 (Autumn): 34-48.
- Cutler, David and Lawrence Summers, 1988, "The Costs of Conflict Resolution and Financial Distress: Evidence from the Texaco-Pennzoil Litigation," *Rand Journal of Economics*, 19 (Summer), 157-172.
- DeAngelo, Harry and Ronald Masulis, 1980, "Optimal Capital Structure Under Corporate and Personal Taxation," *Journal of Financial Economics*, 8 (March), 3-30.
- Gopalakrishnan, V., 1994, "Accounting Choice Decisions and Unlevered Firms: Further Evidence on Debt/Equity Hypothesis," *Journal of Financial and Strategic Decisions*, 7 (Fall, Number 3), 33-47.
- Fama, Eugene and Kenneth French, 1997, "Industry Costs of Equity," *Journal of Financial Economics*, 43, 153-193.
- Hamada, Robert, 1972, "Effect of a Firm's Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance*, 27 (May), 435-452.
- Harvey, Campbell and John Graham, 2001, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics*, 60 (May), 187-243.
- Farrar, D. and L. Selwyn, 1967, "Taxes, Corporate Financial Policy, and Returns to Investors," *National Tax Journal*, 20, 444-454.
- Fischer, Edwin O., Robert Heinkel and Josef Zechner, 1989, "Optimal Dynamic Capital Structure Choice: Theory and Tests," *The Journal of Finance*, 44, 19-40.
- Hull, Robert, 1999, "Leverage Ratios, Industry Norms, and Stock Price Reaction: An Empirical Investigation of Stock-for-Debt Transactions," *Financial Management*, 28 (Summer, Number 2), 32-45.

Hull, Robert and Stuart E. Michelson, 1999, "The Information Contents of Senior Offerings that Reduce Junior Securities," *Quarterly Review of Economics and Finance*, 39 (Fall, Number 3), 419-438.

Jensen, M. and W. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3 (October), 305-360.

Kayhan, Ayla and Sheridan Titman, 2004, "Firms' Histories and Their Capital Structure," NBER Working Paper No. W10526.

Kim, E. Han, 1978, "A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity," *Journal of Finance*, 33 (March), 45-63.

Kim, E. Han, 1982, "Miller's Equilibrium, Shareholder Leverage Clienteles, and Optimal Capital Structure," *Journal of Finance*, 37 (May), 301-320.

KPMG, [February 2004] report on AGL Gas Networks, Ltd. Retrieved Dec. 11, 2004.  
[http://www.ipart.nsw.gov.au/papers/KPMG\\_February\\_04.pdf](http://www.ipart.nsw.gov.au/papers/KPMG_February_04.pdf).

Kraus, Alan and Robert Litzenberger, 1973, "A State-Preference Model of Optimal Capital Structure," *Journal of Finance*, 28 (September), 911-922.

Lally, Martin, 2004, "The Fama-French Model, Leverage, and the Modigliani-Miller Propositions," *The Journal of Financial Research*, 27 (September), 341-349.

Leland, Hayne and David Pyle, 1977, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *Journal of Finance*, 32 (May), 371-388.

Leland, Hayne, 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49 (September), 1213-1252.

Leland, Hayne and Klaus Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51 (July), 261-275.

Leland, Hayne, 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53 (August), 1213-1243.

Miller, Merton, 1977, "Debt and Taxes," *Journal of Finance*, 32 (May), 261-275.

Modigliani, Franco and Merton Miller, 1963, "Corporate Income Taxes and Cost of Capital: A Correction," *American Economics Review*, 48 (June), 433-443.

Modigliani, Franco, 1982, "Debt, Dividend Policy, Taxes, Inflation and Market Valuation," *Journal of Finance*, 37, 255-273.

Myers, S. and N. Majluf, 1984, "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics* 13 (June): 187-221.

Pinegar, J. Michael, and Lisa Wilbrecht, 1989, "What Managers Think of Capital Structure Theory: A Survey." *Financial Management* 18 (Winter), 82-91.

Ross, Stephen, 1977, "The Determination of Financial Structure: The Incentive-Signaling Approach," *Bell Journal of Economics*, 8 (Spring): 23-40.

Ross, Stephen, 1985, "Debt and Taxes and Uncertainty," *Journal of Finance*, 40 (Spring), 637-658.

State One Stockbroking Ltd., [December 2004] report of The Australian Gas Light Company. Retrieved Dec. 17, 2004. <http://www.stateone.com.au/documents/research/agl.pdf>

Warner, J., 1977, "Bankruptcy Costs: Some Evidence," *Journal of Finance*, 26 (May), 337-348.