DERIVATIVES

The derivative of a function \( y = f(x) \) is equal to the rate of instantaneous change in variable \( y \) with respect to variable \( x \).

Can be denoted in a number of different ways:

\[
\begin{align*}
y' & \quad f'(x) & \quad \frac{dy}{dx} & \quad \frac{df(x)}{dx}
\end{align*}
\]

Graphically, it is equal to the slope of the function at a point.

Positive derivative – positive relationship – positive slope
Negative derivative – negative relationship – negative slope
How derivatives help us find the maximum of a function

Slope = 0
It is the maximum!

Positive slope
Negative slope
Four basic rules of differentiation.
("Differentiation" is the procedure of finding the derivative of a function.)

**Rule 1.** The derivative of a constant is zero.
(Easy to see if you plot it; the derivative is the slope of the graph)
Rule 2. The derivative of a power function (a function of the form $y = x^n$, where $n$ is any number).

If $y = x^2$, then $\frac{dy}{dx} = 2x$

If $y = x^3$, then $\frac{dy}{dx} = 3x^2$, etc.

Overall, $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$, or in other words...

- The power in the original function becomes the coefficient in the derivative;
- The power of $x$ in the derivative is one less than in the original function.
Rule 3. A constant times a function.

If $y = k \ f(x)$, then $\frac{dy}{dx} = k \frac{df(x)}{dx}$

The derivative of a constant times a function is equal to the constant times the derivative of the function.

Example:

$Y = 3 \ x^3 \quad \frac{dY}{dx} =$
Rule 3. A constant times a function.

If \( y = k \ f(x) \), then
\[
\frac{dy}{dx} = k \ \frac{df(x)}{dx}
\]

The derivative of a constant times a function is equal to the constant times the derivative of the function.

Example:
\[
Y = 3 \ x^3 \quad \frac{dY}{dx} = 3 \cdot 3x^2 = 9x^2
\]
Finally, **Rule 4.**

The derivative of the sum of two functions is the sum of their derivatives.

The derivative of the difference of two functions is the difference of their derivatives.

Example:

\[ Y = x^3 - 2x + 3 \]

\[ \frac{dY}{dx} = \]
Finally, **Rule 4.**

The derivative of the sum of two functions is the sum of their derivatives.

The derivative of the difference of two functions is the difference of their derivatives.

Example:

\[ Y = x^3 - 2x + 3 \]

\[ \frac{dY}{dx} = 3x^2 - 2 \]
Several problems for individual work:

i. Find the derivative of the following function with respect to variable x:

\[ Y = 2x^3 - 14x^2 + 3x - 100 \]

ii. Find the derivative with respect to x and find the positive value of x that maximizes the value of function Y. (Recall this can be done by setting the derivative equal to zero and solving it for x.) Confirm your result in Excel.

\[ Y = x - x^3 + 1 \]
iii. (The more general case)
Find the derivatives of the following expressions. Treat anything that is not x as you would treat a constant, or a number.

Y = Ax² + Bx + C

Y = c·x – d