Washburn University Math Day Exam
November 9, 2004

1. Simplify: $5-\left(7(2-5)-3^2\right)$

   A) -30 B) 35 C) 17 D) -25 E) -7

2. The least common multiple of 18 and 40 is:

   A) 160 B) 240 C) 360 D) 480 E) 720

3. Factor completely: $12x^2 - x - 63$

   A) $(2x - 9)(6x + 7)$ B) $(12x + 21)(x - 3)$ C) $(4x + 9)(3x - 7)$

   D) $(6x - 9)(2x + 7)$ E) cannot be factored

4. Expand: $x(5x^2 + 2)(3x - 1)$

   A) $15x^4 + x^3 - 2x$ B) $15x^3 - 5x^2 + 6x - 2$ C) $5x^4 - 5x^3 + 2x^2 - 2x$

   D) $15x^4 - 5x^3 + 6x^2 - 2x$ E) $15x^3 + x^2 - 2x$

5. Simplify: $3t[4 - (t - 3)] + t(t + 5)$

   A) $t^2 + 23t$ B) $6t - 2t^2$ C) $26t - 2t^2$

   D) $41t - 11t^2$ E) $3t^3 + 12t^2 + 21t$

6. Solve the equation, if possible. $16 + 4[5x - 4(x + 2)] = 7 - 2x$

   A) $x = \frac{-1}{6}$ B) $x = \frac{23}{2}$ C) $x = \frac{-41}{6}$ D) $x = \frac{23}{6}$ E) No solution

7. Simplify the expression using only positive exponents. (Assume all variables denote non-zero numbers.) $\left(\dfrac{6u^2v^{-1}}{27u^{-1}v^3}\right)^{-2}$

A) $\dfrac{81v^8}{4u^6}$ B) $\dfrac{2916v^8}{36u^6}$ C) $\dfrac{81v^4}{4u^2}$ D) $-\dfrac{36u^6}{2916v^8}$ E) $-\dfrac{81v^4}{4u^2}$

8. Combine and simplify (for all x in the domain): $\dfrac{x+2}{x-1}-\dfrac{2}{x+6}-\dfrac{14}{x^2+5x-6}$

A) $\dfrac{x(x+4)}{(x-1)(x+6)}$ B) $\dfrac{-13x^2-64x+98}{(x-1)(x+6)}$ C) $\dfrac{x-1}{x+6}$

D) $\dfrac{x^2+6x-4}{(x-1)(x+6)}$ E) $\dfrac{x}{x-1}$

9. Perform the indicated division. $\dfrac{x^3-5x+3}{x-2}$ (for $x \neq 2$).

A) x^2-3x-3 B) $x^2-7x+14-\frac{25}{x-2}$ C) $x^2-3x-6-\frac{9}{x-2}$

D) $x^2-2x-1+\frac{5}{x-2}$ E) none of these

10. Solve the equation for x: $6x^3 = 3x - 7x^2$

A) $x = -3, 0, 1/6$ B) $x = -1/3, 0, 3/2$ C) $x = -1, 0, 1/2$

D) $x = -1/2, 0, 1$ E) $x = -3/2, 0, 1/3$

11. John has 20 coins consisting of only dimes and quarters. The value of the coins is \$3.20. How many dimes does he have?

A) 6 B) 8 C) 10 D) 12 E) 15

12. A father is 7 times as old as his son is now. Five years from now, the father will be 4 times as old as his son will be. How old will the father and son be (in that order) 4 years from now?

A) 35, 5 B) 39, 9 C) 44, 14 D) 32, 8 E) 40, 10

13. The degree measures of the angles of a triangle are all integer values. If A, B, and C represent the degree measures and A < B < C, what is the least possible value of C?

A) 59^o B) 60^o C) 61^o D) 62^o E) 177^o

14. If the average of 5 consecutive integers is 12, what is the sum of the least and greatest of the 5 integers?

A) 24 B) 14 C) 12 D) 11 E) 10

15. A half of a third of a fourth of a Liang is equal to a fifth of a sixth of seven Chi's. How many Liang's does it take to make eight Chi's?

A) 1/630 B) 5/28 C) 32/35 D) 10/7 E) 176/5

16. Give the equation of the ellipse with center at $(5, -3)$, a focus at $(9, -3)$ and length of minor axis 6.

A) $\frac{(x+5)^2}{16} + \frac{(y-3)^2}{9} = 1$ B) $\frac{(x-5)^2}{16} + \frac{(y+3)^2}{9} = 1$

C) $\frac{(x+5)^2}{16} - \frac{(y-3)^2}{9} = 1$ D) $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{9} = 1$

E) $\frac{(x+5)^2}{11} + \frac{(y-3)^2}{36} = 1$

17. Solve the equation: $\sqrt{x+5} - \sqrt{x} = 1$.

A) $x = 4$ B) $x = 1$ C) $x = 4, 1$ D) $x = 9$ E) No solution

18. The rational function $f(x) = \frac{2x^2 - x - 1}{x + 3}$ has oblique asymptote:

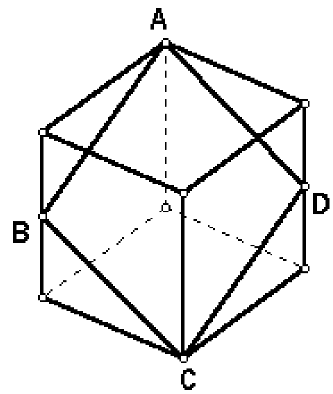
A) $y = 2x - 1$ B) $y = 2x - 1/3$ C) $y = 2x - 7$

D) $y = 2x - 5$ E) $f(x)$ does not have an oblique asymptote

19. A triangle has side lengths of 50 cm, 120 cm and 130 cm. What is the shortest altitude of the triangle?

A) 50 cm B) 600/13 cm C) 156/5 cm D) 325/6 cm E) $25\sqrt{2}$ cm

20. The cube below has edges that measure 10 units in length. Points B and D are midpoints of edges of the cube. Find the perimeter of quadrilateral ABCD.



A) $10\sqrt{3}$ units B) 48 units C) $20\sqrt{3}$ units D) $20\sqrt{5}$ units E) 40 units

21. The decimal number 2004 has base eight (octal) representation:

A) 3722_{eight} B) 3724_{eight} C) 3726_{eight} D) 3730_{eight} E) none of these

22. Solve the equation for x in terms of y. $x^2y + 2yx - x = 2$

A) $x = \frac{1}{y}, -2$ B) $x = x^2y + 2yx - 2$ C) $y = \frac{1}{x}$

D) $x = \frac{-2y+1 \pm \sqrt{4y^2 - 12y + 1}}{2y}$ E) $x = 0, \frac{-2y-1}{y}$

23. A fenced backyard is in the shape of a regular octagon. The perimeter of the fence is 112 feet. A dog is tied to an inside corner of the fence with a 10 foot chain. How much roaming area inside the fence does the dog have?

A) $\frac{100p}{3}$ ft^2 B) $(50\sqrt{2} + 25)$ ft^2 C) 125 ft^2 D) $24\sqrt{2}$ ft^2 E) $\frac{75p}{2}$ ft^2

24. How many solutions are there to the equation: $\left| |x - 1| - 7 \right| = 3$?

A) 0 B) 1 C) 2 D) 3 E) 4

25. Which of the following is a function that is both increasing and negative for all real *x*?

A) $f(x) = 2^x$ B) $f(x) = 2^{-x}$ C) $f(x) = -2^x$ D) $f(x) = -2^{-x}$ E) none of these

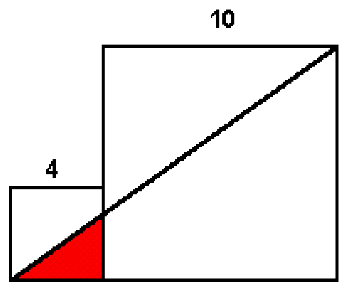
26. Solve the system of equations:

$$2x + y - z = -2$$
$$x + 2y - z = -9$$
$$x - 4y + z = 1$$

A) $x = 6, y = -1, z = 13$ B) $x = 0, y = 4, z = 6$ C) $x = -1, y = 0, z = 8$

D) The system has no solution. E) The system has a solution, but the solution is not listed.

27. The figure below depicts two squares with side lengths 4 inches and 10 inches respectively. As pictured, there is a straight line connecting opposite corners of the squares. Find the area of the shaded region.



A) $4\sqrt{2}\ \text{in}^2$ B) $\frac{5}{2}\ \text{in}^2$ C) $6\ \text{in}^2$ D) $\frac{40}{7}\ \text{in}^2$ E) none of these

28. Find the product of all solutions to the trigonometric equation $2\sin q \cos q = \cos q$ where $0 £ q £ 2p$.

A) 0 B) $\frac{5p^4}{24}$ C) $\frac{5p^2}{36}$ D) $\frac{5p^4}{48}$ E) $\frac{2p^2}{9}$

29. A function *f*(*x*) has vertical asymptote $x = 4$ and horizontal asymptote $y = -2$. Find the asymptotes of $h(x)$ if $h(x) = -3f(x+1)$.

A) $x = 5,\ y = -2$ B) $x = 3,\ y = -5$ C) $x = 5,\ y = -6$

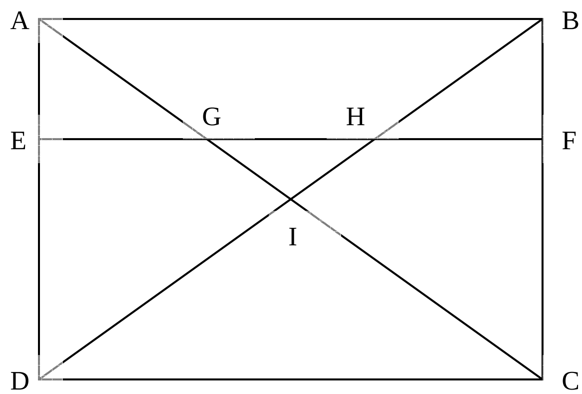
D) $x = 3,\ y = 6$ E) $x = 4,\ y = -2$

30. The perpendicular lines $y-3=\frac{1}{4}(x-3)$ and $y-3=-4(x-3)$ are tangent to a circle at (-1, 2) and (4, -1) respectively. The equation of the circle is:

A) $x^2 + (y + 2)^2 = 34$ B) $x^2 + (y + 2)^2 = 17$ C) $(x - 3/2)^2 + (y - 1/2)^2 = 17/2$

D) $(x - 3)^2 + (y - 3)^2 = 17$ E) not enough information

31. Figure ABCD below is a rectangle. Line segment EF is parallel to AB and intersects the diagonals of the rectangle at points G and H as shown. How many triangles in the figure are similar to triangle AEG (not counting triangle AEG itself)?



A) 1 B) 3 C) 4 D) 5 E) more than 5

32. Let x and y be positive real numbers satisfying the equation $\frac{x-3y}{x+11y}=\frac{y}{x+3y}$. Then, the ratio of x to y (x:y) is:

A) 3:1 B) 3:2 C) 4:1 D) 5:1 E) none of these

33. Solve the absolute value equation for x: $|2x-5| > |3x+2|$

A) $x > -7$ B) $x < 3/5$ C) $x > -2/3$ D) no solution E) none of these

34. Radioactive Sarahium has a half-life of 34 years (that is, half the original amount remains after 34 years). A Sarahium contamination will be considered safe once there is less than one one-thousandth of the original Sarahium left; this will take approximately how many years?

A) 102 B) 204 C) 340 D) 680 E) 1,000

35. An arithmetic sequence begins as follows: 11, $2x - 3y$, $x - 2y$, $4y - 3x + 1$, $5y - 4x + 1$,…. Find the value of $x + y$:

A) –7 B) 7 C) 17 D) 27 E) not enough information given

36. Regular hexagon ABCDEF has area one square unit. The area of rectangle ABDE (in square units) is:

A) 1/2 B) 2/3 C) $\frac{\sqrt{2}}{2}$ D) $1-\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{2}$

37. Find the number of solutions to the following system, where x and y are restricted to $0 \text{ £ } x < 2p$ and $0 \text{ £ } y < 2p$.

$$\sin(x)+\sin(y)=1/2$$
$$\cos(x)-\sin(y)=1/2$$

A) 0 B) 2 C) 4 D) 6 E) 8

38. Recall that "n factorial" means $n! = n(n-1)(n-2)\ldots(3)(2)(1)$, and that $0! = 1$. Let n and k be nonnegative integers. Which of the following must be true?

(i) If $n \geq k$, then $\frac{n!}{k!}$, $\frac{n!}{(n-k)!}$, and $\frac{n!}{k!(n-k)!}$ are all integers.

(ii) If p is prime, then $p! + 1$ is prime.

(iii) $(nk)! = n!k!$

(iv) $\sum_{k=0}^{n} \frac{n!}{k!(n-k)!} = 2^n$

(v) $C(n,k)+C(n,k-1)=C(n+1,k)$, for $n > k$, where $C(n,k)$ represents the combination number "n choose k."

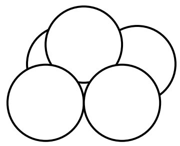
A) (i), (ii) and (iii) only B) (i), (iii) and (iv) only C) (i), (iii) and (v) only

D) (i), (iv) and (v) only E) all are true

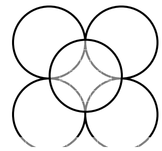
39. A floor is tiled with 1 inch by 1 inch squares. The floor requires exactly 9 tiles horizontally and exactly 14 tiles vertically. How many squares do the tiles form? (Count all 1 by 1 squares, all 2 by 2 squares, etc).

A) 144 B) 616 C) 456 D) 384 E) 510

40. A pyramid is to be made from 5 spheres, with 4 spheres forming the base of the pyramid and one sphere on top as pictured below. Each sphere on the base is tangent to two other spheres on the base. Each sphere measures 2 inches in diameter. How many inches high is the pyramid?



(View from the front of the pyramid)



(View from the top of the pyramid.)

A) $2+\sqrt{2}$ B) $5-2\sqrt{2}$ C) $4-\sqrt{2\sqrt{2}-2}$ D) $4-\sqrt{2}$ E) $2+\sqrt{3}$