Washburn University Math Day Exam
November 3, 2009

1. Evaluate $\frac{2-4\cdot 6}{8-10}=$

A) 12 B) -11 C) -14 D) 6 E) 11

2. Find the product. Give your answer in lowest terms. $\frac{32}{15}\cdot\frac{35}{24}$

A) $\frac{28}{15}$ B) $\frac{7}{6}$ C) $\frac{16}{3}$ D) $\frac{8}{3}$ E) $\frac{28}{9}$

3. Evaluate $-5^2+20\div 5-1=$

A) -22 B) 28 C) 224 D) -2 E) 30

4. Perform the indicated operation and simplify: $4-2(3-5x)$ =

A) $10x-2$ B) $6-10x$ C) $4+4x$ D) $5x-2$ E) $-2-10x$

5. Perform the indicated operation and simplify: $\left(x^2+3xy+7y^2\right)-\left(y^2+yx-11x^2\right)$

A) $12x^2+4xy+6y^2$ B) $12x^2+2xy+6y^2$ C) $-10x^2+4xy+6y^2$

D) $10x^2-4xy-8y^2$ E) none of these

6. Simplify: Leave no negative exponents in your answer. $\frac{\left(4xy^{-2}\right)^{-2}}{2xy^3}$

A) $\frac{y}{32x^3}$ B) $-\frac{8y}{x^3}$ C) $\frac{1}{32x^3y^7}$ D) $\frac{y}{8x}$ E) $-\frac{4}{x^3y^{-7}}$

7. Give the coordinates of the point located 7 units to the left and 4 units above the point $(4,-2)$.

A) $(8,-9)$ B) $(0,-5)$ C) $(11,2)$ D) $(-3,-6)$ E) $(-3,2)$

8. Find the difference. Write the answer in reduced form. $\frac{1}{15}-\frac{5}{21}$

A) $\frac{2}{7}$ B) $-\frac{6}{35}$ C) $-\frac{4}{105}$ D) $\frac{2}{3}$ E) $-\frac{1}{18}$

9. If $a = -3$ and $b = -2$, find the value of the expression $b^2 - 4a$:

A) -17 B) 8 C) -3 D) -8 E) 16

10. Perform the indicated operation and give the answer in scientific notation: $\dfrac{\left(5\times10^{4}\right)\left(8\times10^{-9}\right)}{2\times10^{7}}$

A) 2×10^{-11} B) 2×10^{-13} C) 2×10^{2} D) 2×10^{-12} E) 2×10^{3}

11. Solve: $\frac{1}{2}x+\frac{1}{4}=\frac{1}{4}(x-6)$

A) $x=4$ B) $x=\frac{-3}{4}$ C) $x=-7$ D) $x=\frac{25}{2}$ E) $x=-12$

12. Solve: $|5x-1|\le 8$

A) $-\frac{9}{5}\le x\le\frac{9}{5}$ B) $-\frac{7}{5}\le x\le\frac{9}{5}$ C) $-\frac{9}{5}\le x\le\frac{7}{5}$

D) $x\le-\frac{7}{5},\quad x\ge\frac{9}{5}$ E) no solution

13. Find the degree of the polynomial: $3x\left(2-4x^4\right)^8\left(x^3+9x-1\right)^5$

A) 32 B) 48 C) 21 D) 14 E) 47

14. If there are 5 pips in a stet, 3 goks in a stet, 2 stets in a drak, and 7 huls in a drak, how many huls are in 8 pips?

A) $\frac{28}{5}$ B) $\frac{60}{7}$ C) $\frac{56}{5}$ D) $\frac{80}{7}$ E) $\frac{56}{15}$

15. A line with slope $-\frac{3}{5}$ passes through the point $(4,-2)$. Find the *x*-coordinate of the point on the line that has a *y*-coordinate of 13.

A) $x=\frac{17}{5}$ B) $x=-\frac{43}{3}$ C) $x=13$ D) $x=-21$ E) $x=-5$

16. A card is selected at random from a standard 52 card deck. Assuming all cards are equally likely to be selected, what is the probability that a red card or a king is selected?

A) 15/26 B) 7/13 C) 17/26 D) 4/13 E) 27/52

17. The graph of $x^2 + y^2 - 4x + 6y + 13 = 0$ is a:

A) pair of straight lines B) parabola C) circle D) single point E) hyperbola

18. For $0^\circ \le \theta \le 360^\circ$, how many solutions are there to $2\sin^2\theta - \sin\theta = 1$?

A) 0 B)1 C) 2 D) 3 E) 4

19. Let $f(x) = \dfrac{3x+1}{x-2}$. Find $f^{-1}(3)$.

A) -2 B) 3 C) 10 D) 1/10 E) undefined

20. If *n* is a positive integer, then the expression $\dfrac{2^{n-2}(2n+1)!}{n2^{2n-1}(2n-1)!}$ simplifies to:

A) $\dfrac{(2n+1)}{(2n-1)2^{n+1}}$ B) $\dfrac{(2n+1)n}{2^{n-1}}$ C) $\dfrac{(2n-1)}{2^{n+2}}$ D) $\dfrac{(2n+1)}{2^n}$ E) $\dfrac{2n}{n2^n}$

21. In a 2008 survey, a poll was conducted of male and female Americans. Of those polled, 41% were female, 68% believed that the nation was ready for a female president, and 25% were female **and** believed the nation was ready for a female president. What percentage of Americans polled were male **and** did not believe the nation was ready for a female president?

A) 75% B) 32% C) 91% D) 16% E) 34%

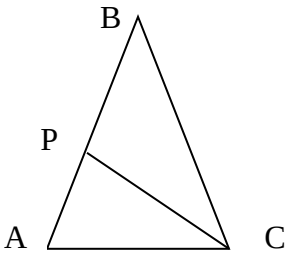
22. Simplify the trigonometric expression $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta}$.

A) $\csc\theta$ B) $\sec\theta$ C) $\sin\theta$ D) $\cos\theta$ E) $\tan\theta$

23. The circle $(x-3)^2 + (y+1)^2 = 25$ and the line $y = \frac{1}{2}x - 5$ intersect in two points. Find the midpoint of the line segment between these two intersection points.

A) (5,3) B) (3,–1) C) (3.5,–2.5) D) (4,–3) E) (4.5, 4)

24. In triangle *ABC*, $\angle A = \angle C$. Point *P* on $\overline{AB}$ is such that *PB* =*PC* = *AC*. Find the measure of angle *B*.



A) 30° B) 48° C) 36° D) 24° E) 60°

25. Evaluate: $\log_3\left[\left(\log_8 2\right)^{\log_2 8}\right]$

A) -3 B) -1/3 C) 1/3 D) 3 E) 9

26. Consider the following statements:

All doodads are flibertigibbits.
No flibertigibbits are whatchamacallits.
If a whozamawhatsis is a flibertigibbit, then it isn't a doodad.

Based on the above statements, determine which of the following is/are true.

I. Some doodads are whatchamacallits.
II. Some doodads are whosamawhatsises.

A) I and II B) I only C) II only
D) neither I nor II E) There is not enough information to decide.

27. In a certain community there are 100 married couples. Two-thirds of the husbands who are taller than their wives are also heavier than their wives. Three-fourths of the husbands who are heavier than their wives are also taller. Twelve wives are both heavier and taller than their husbands. How many husbands are both taller and heavier than their wives?

A) 36 B) 48 C) 60 D) 72 E) 84

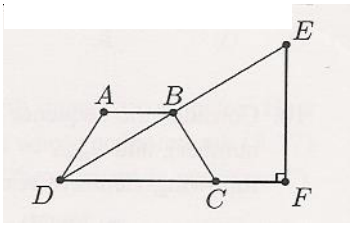
28. Recall that 2_{nine} denotes the number 2 base 9. As a decimal (base 10) numeral, $\sqrt{150_{eight} + 25_{six}}$ simplifies to:

A) 10 B) 11 C) 12 D) 13 E) 15

29. Suppose a committee of 3 people is to be randomly selected from a club that has 9 men and 7 women members. What is the probability that the committee has at least one woman?

A) 13/40 B) 7/9 C) 17/20 D) 7/16 E) 3/16

30. In the figure, *ABCD* is an isosceles trapezoid with side lengths *AD* = *BC* = 5, *AB* = 4, and *DC* = 10. The point *C* is on $\overline{DF}$ and *B* is the midpoint of hypotenuse $\overline{DE}$ in the right triangle *DEF*. Find the length of $\overline{EF}$.



A) $5\sqrt{2}$ B) $4\sqrt{2}$ C) 8 D) $3\sqrt{6}$ E) 12

31. Recall that $i = \sqrt{-1}$. Give the *x*-value for the solution to the following complex linear system. Note that the solution can itself involve complex numbers.

$$2ix-(3-i)y=i$$
$$(1+i)x+(3-i)y=1-i$$

A) $\frac{1}{10}-\frac{3}{10}i$ B) $\frac{1}{4}-\frac{4}{4}i$ C) $\frac{1}{10}+\frac{3}{10}i$ D) $\frac{1}{4}+\frac{4}{4}i$ E) The system has no solution.

32. *A, B, C, a, b,* and *c* are positive integers such that $cA < aC < bA < aB < cB < bC$. Arrange the rational numbers $\frac{a}{A}$, $\frac{b}{B}$, and $\frac{c}{C}$ from least to greatest.

A) $\frac{a}{A},\frac{b}{B},\frac{c}{C}$ B) $\frac{c}{C},\frac{a}{A},\frac{b}{B}$ C) $\frac{b}{B},\frac{a}{A},\frac{c}{C}$ D) $\frac{b}{B},\frac{c}{C},\frac{a}{A}$ E) none of these

33. A rope the length of the circumference of the earth (about 40,075.02 km) is wrapped around the earth at the equator (assume the equator is a circle). If one meter of rope is added, and the new rope is lifted off the earth at a uniform height, the distance the rope will be off the ground is closest to which of the following numbers?

A) 10 cm = 10^{-1} m B) 1 cm = 10^{-2} m C) 1 mm = 10^{-3} m
D) 1 μm = 10^{-6} m E) 1 nm = 10^{-9} m

34. If $a+b=1$ and $a^2+b^2=2$ then a^3+b^3 equals:

A) 4 B) 7/2 C) 3 D) 5/2 E) 2

35. Bill started with 140 coins consisting of nickels, dimes and quarters, worth $10. He spent $3 of his money, consisting of a dollar's worth of quarters and a combination of nickels and dimes where twice as many nickels as dimes were spent. The remaining $7 had as many dimes as there were quarters originally. Find the number of dimes in the original 140-coin collection.

A) 25 B) 20 C) 15 D) 10 E) not enough information

36. Two busses are initially 1,000 miles apart. Both busses start at the same time, heading directly towards each other, bus A is moving at 50 miles per hour and bus B at 60 miles per hour. After 3 hours, bus A stops for an hour. One hour after bus A has resumed travel, bus B stops for one half hour. Two hours after bus B has resumed travel, bus A stops again for one hour. Assuming no more stops take place, how much time passes from the time the two busses embarked on their respective journeys to when they meet?

A) 9 ½ hours B) 9 $\frac{7}{8}$ hours C) 10 $\frac{1}{11}$ hours
D) 10 $\frac{3}{11}$ hours E) cannot be determined from the information given

37. How many integers between 1 and 2009 are divisible by 3, 4, or 5?

A) 1172 B) 1205 C) 1139 D) 1572 E) 1539

38. Which of the following is equivalent to the continued fraction?

$$a+\cfrac{b}{2a+\cfrac{b}{2a+\cfrac{b}{2a+\cdots}}}$$

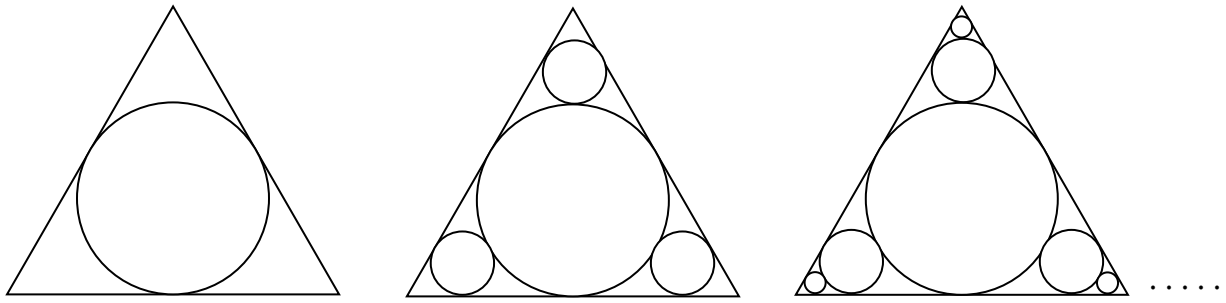
A) $\sqrt{a^2+b}$ B) $a+\sqrt{a^2+b}$ C) $a-\sqrt{a^2+b}$

D) $\frac{3a}{2}+\sqrt{a^2+b}$ E) $2a+\sqrt{b}$

39. An impatient woman walks all the way up an up-going escalator. She takes 20 steps when she walks at the rate of one step per second. An even more impatient man takes 32 steps on the same escalator but his rate is two steps per second. How many (visible) steps does the escalator have?

A) 20 B) 40 C) 60 D) 80 E) 100

40. A circle of radius 1 inch is inscribed in an equilateral triangle. At each vertex of the triangle, a smaller circle is inscribed which is tangent to the larger circle and both sides of the triangle at the corresponding vertex. This process continues indefinitely with increasingly smaller circles inscribed at each vertex of the original triangle (see pictures). What is the sum of the circumferences of all circles?



A) 5π inches B) $\frac{14\sqrt{3}-1}{2\sqrt{3}-1}\pi$ inches C) 8π inches D) $\frac{4\sqrt{3}}{2\sqrt{3}-1}\pi$ inches E) 3π inches